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ANALYSIS OF AEROSOL AGING IN THE ROTATING DRUM CHAMBER

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RESEARCH AND TECHNOLOGY DIRECTORATE

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14. ABSTRACT Two of the most recent theories describing aerosol mechanics in the rotating drum chamber were compared with experimental measurements. Both theories predicted nearly the same results but did not match out measurements. Possible causes for the discrepancies were investigated, and it was found that convective diffusion accounts for the difference with the stirred settling model predicting measurements reasonably in the limit where rotation rate becomes zero. Once rotation begins, transport by diffusion was found to dramatically decrease, probably because convective currents gyrate driving smaller eddies that experience rapid viscous dissipation. Damping of convective diffusion was found to be sufficient for transport to be dominated by centrifugal acceleration three orders of magnitude smaller than gravity acting on particles several microns in diameter. Finally, the discrepancies at nonzero rates of rotation were explained by triboelectric charging during dissemination, which can normally be ignored during static chamber tests governed by convective or strong turbulent diffusion transport up to a laminar boundary layer where gravitational settling dominates. When the drum was rotated, transport by the repulsive coulombic force between a triboelectric charged particle and the monopolar charged aerosol cloud dominated transport by centrifugal acceleration.

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PREFACE

The work described in this report was authorized under Project No. 622384/ACB2. This work was started in September 2003 and completed in October 2003.

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ANALYSIS OF AEROSOL AGING IN THE ROTATING DRUM CHAMBER

1. INTRODUCTION

The rotating drum aerosol chamber was developed to maintain aerosol concentrations over long periods of time in order to study aerosol aging. By rotating the drum about a horizontal cylindrical axis we can avoid gravitational settling which removes significant fractions of the aerosol from static aerosol chambers over times that are large compared to the chamber height divided by the Stokes settling velocity. The rotating drum is useful for studying the viability of biological aerosols over long times and for studying forces and effects that normally would be overwhelmed by gravitational settling. Two theoretical treatments of aerosol mechanics in a rotating drum, (Gruel et al. 1987) and (Asgharian and Moss 1992), will be compared to one another and referred to as Gruel's theory and Moss's theory. Then predictions from these theories will be compared to recent rotating drum chamber measurements (Sutton 2003). Attempts will be made to explain any significant differences between measurements and predictions using these theories.

2. EXPERIMENTAL DETAILS

In the experimental study (Sutton 2003) aluminum oxide aerosol was introduced into a 1 meter diameter by 1 meter long cylindrical chamber (drum) from a large plastic trash bag containing aerosol disseminated pneumatically using the SRI sonic nozzle. This was done, starting with nearly zero initial chamber volume, by moving the drum piston wall away from the opposite wall creating negative pressure that pulled the aerosol from the bag into the drum through a section of Tygon tubing attached to the ½ inch diameter port located at the center of the opposite drum wall on the drum axis. Aerosol samples were taken to determine aerosol concentration and particle size as a function of time, rotation rate and sampling port location by moving the drum piston wall toward the opposite wall creating positive pressure. Small volumes of aerosol were pushed through one of four ½ inch diameter sampling ports, in the opposite wall located at radial positions between the axis and 5 cm from the curved wall, into an Aerodynamic Particle Sizer manufactured by TSI. Essentially the chamber was filled and emptied like a syringe.

3. THEORETICAL BASIS

Gruel's theory describes aerosol particle spiral trajectories in the laboratory frame of reference due to combined gravitational and centrifugal forces. A suspension volume is defined for each particle size as the fraction of aerosol remaining after one revolution of the chamber has cleared away all aerosol within a distance of one radius of gyration of the particle. This is best understood in the rotating drum frame of reference where particles gyrate due to rotating gravitational force and intercept the wall if within a distance of one radius of gyration. As they gyrate due to gravity they slowly move radially out toward the drum wall due to centrifugal force following spiral paths in the laboratory frame of reference and flat helical paths

with small pitch angles due to the relatively small centrifugal force compared to gravitational force in the rotating drum frame of reference. Gruel's theory is appropriate when centrifugal force is much less than gravitational force predicting aerosol total number suspended after one revolution and decay thereafter as aerosol particles move into the clear zone due to centrifugal force.

Moss's theory uses the same equations of motion but introduces the concept of limiting trajectory to describe the combined effects of gravity and centrifugation over the complete range of rotation rates. Moss's theory predicts aerosol concentration decay beginning when drum rotation starts.

4. RESULTS AND ANALYSIS

The radius of gyration of a particle in the rotating drum chamber is

$$r_0 = \frac{\tau g}{\omega} \tag{1.1}$$

where τ, g *and* ω are respectively the particle relaxation time, acceleration due to gravity and angular velocity of drum rotation. The relaxation time can be expressed

$$\tau = \frac{\rho d^2 C_c}{18\eta} \tag{1.2}$$

in terms of η, ρ, d *and* C_c which are respectively the dynamic viscosity of air, the particle density, diameter and Cunningham slip correction factor

$$C_c = 1 + Kn\left[1.257 + 0.4\exp\left(-\frac{1.1}{Kn}\right)\right] \tag{1.3}$$

where $Kn = 2\lambda/d$ is the Knudsen number which is twice the ratio of the mean free path of air molecules, 0.071 μm at standard temperature and pressure, over the particle diameter. The total number of aerosol particles suspended in the chamber decays exponentially according to Gruel's theory so that the fraction of aerosol remaining is

$$\frac{N}{N_0} = \left[\frac{(R - r_0)^2}{R^2}\right]\exp\left(-2\tau\omega^2 t\right) \tag{1.4}$$

where R *and* t represent the drum radius and time **beginning after one revolution**. Equivalent expressions for the fraction of aerosol remaining based on Moss's theory describe what happens

during the first revolution and include the possibility that centrifugal acceleration exceeds gravitational acceleration

$$\frac{N}{N_0} = \exp\left(-2\tau\omega^2 t\right) \quad for \quad \frac{H}{R} \le 1 - \exp\left(-\tau\omega^2 t\right)$$

$$\frac{N}{N_0} = \frac{1+\exp\left(-2\tau\omega^2 t\right)}{2} - \frac{1}{\pi}\left[\begin{array}{l} \frac{H_1}{R}\sqrt{1-\left(\frac{H_1}{R}\right)^2} + \sin^{-1}\frac{H_1}{R} + \\ \frac{H_2}{R}\sqrt{\exp(-2\tau\omega^2 t) - \left(\frac{H_2}{R}\right)^2} + \\ \exp(-2\tau\omega^2 t)\sin^{-1}\left(\frac{H_2}{R}\right) \end{array}\right] \quad for \left\{\begin{array}{l} 1-\exp\left(-\tau\omega^2 t\right) \le \frac{H}{R} \le \\ 1+\exp\left(-\tau\omega^2 t\right) \end{array}\right\} \quad (1.5)$$

$$\frac{N}{N_0} = 0 \quad for \quad \frac{H}{R} \ge 1 + \exp\left(-\tau\omega^2 t\right)$$

where

$$\begin{aligned} H &= \frac{\tau g}{\omega}\sqrt{1-2\cos(\omega t)\exp(-\tau\omega^2 t)+\exp(-2\tau\omega^2 t)} && for\ \omega t < \pi \\ H &= \frac{\tau g}{\omega}\left[1+\exp(-\tau\omega^2 t)\right] && for\ \omega t \ge \pi \\ H_1 &= \frac{R^2 + H^2 - R^2\exp(-2\tau\omega^2 t)}{2H} \\ H_2 &= H - H_1 \end{aligned} \qquad (1.6)$$

and t here represents the time **beginning when the first revolution begins.**

The fraction of aerosol remaining suspended in the drum based on Moss's theory is surface plotted in Figures 1 through 13. The radius of gyration and fraction of aerosol remaining after one revolution based on Gruel's theory is surface plotted in Figures 14 and 15. The ratio of Gruel's solution divided by that of Moss predicting total aerosol mass or number is surface plotted in Figures 16 through 28 after compensating for the 1/rpm time offset by assuming aerosol is removed at a constant rate during the first revolution to create a clear zone. It should be pointed out that these expressions for total number of aerosol particles suspended in the chamber can be used to predict total aerosol mass suspended and to predict aerosol concentration that remains homogeneous. If the theoretical predictions of total number of aerosol particles in the chamber are to be compared to concentration measurements taken from a single port location, the concentration profile within the chamber must be known.

The rotating drum used in the experiment is shown in Figure 29 and was loaned to us by Owen Moss. The drum has a 750 liter capacity and four sampling ports located in the drum wall opposite the piston wall are shown in Figure 30. Port 1 is located 5 cm from the outer

curved wall of the chamber and ports 2, 3, and 4 are located 14, 24, and 50 cm from that wall with port 4 on the drum axis. Aerosol concentration spatial distribution can be monitored by comparing data taken from these four ports and Figure 31 shows that the initial concentration and size distribution are homogeneous. Figures 32 through 36 summarize these measurements for 1 μm through 5 μm aerodynamic diameter particles respectively. In each of these figures the data are grouped according to rotation rates of 0, 1, 2, 4, 6, and 8 rpm indicated along the x axis at the time of measurement. The ratios of number concentrations measured through ports 1, 2, and 3 over those measured through port 4 (the axis port) are shown on the y axis and define the error bars at times t = 0, 2, 4, and 6 hours. Concentrations appear to be fairly homogeneous except for the 6 rpm test after 4 hours with axis port concentration slightly higher than the other three ports. This is to be expected considering that positive pressure in the drum forces the aerosol through the port from radial positions surrounding the port so that the existence of a clear zone, even at port 1 near the wall, will not be evidenced by anything like a step function change in concentration.

Concentration fraction measurements taken through the drum axis port are plotted at all rates of rotation as "percent of initial particles" in Figures 37 through 42 for respectively 1, 2, 3, 4, 5, and 6 μm aerodynamic diameter aerosol particles. Corresponding plots based on Moss's theory predictions are shown in Figures 43 through 48. Significant differences between predictions and measurements are obvious especially at rpm=0 where Moss's theory greatly underestimates suspension times. The stirred settling model is appropriate not only for turbulent mixing due to fans but also convective mixing due to small differences between air and chamber wall temperatures generally encountered with static chambers (Fuchs 1964). The stirred settling model assumes that convective or turbulent diffusion maintains uniform concentration throughout the chamber volume up to the laminar boundary layer near the wall. Within the laminar boundary layer diffusion is damped and Stokes settling becomes the dominate mode of particle transport to the wall where it is deposited reducing total aerosol particle number in accordance with the equation

$$\frac{N}{N_0} = \exp\left(-\frac{2vt}{\pi R}\right) \tag{1.7}$$

for a tube or cylindrical chamber lying on its side where $v = \tau g$ is the Stokes settling velocity. Measurements at rpm = 0 are summarized in Figure 49 and the stirred settling model predictions appearing in Figure 50 are a good match.

Measured aerosol decay rates are dramatically reduced when the drum is rotated suggesting that convective diffusion is reduced because transport near the wall due to gravitational gyration is not much different from that due to gravitational settling in a static chamber. Convective motion of buoyant air in the chamber can be expected to involve gyrations that drive progressively smaller eddies until viscosity damps out the process and drum rotation will smooth out temperature gradients in the drum wall. If convective diffusion is the dominant mechanism controlling transport, then we can estimate what happens by applying the solution for diffusion inside a spherical chamber toward a perfectly absorbing walls and obtain

$$\frac{N}{N_0}=\frac{6}{\pi^2}\sum_{j=1}^{\infty}\frac{\exp\left(D\pi^2 j^2 t/R^2\right)}{j^2} \tag{1.8}$$

where D *and* R are respectively the convective diffusion coefficient and chamber radius (Fuchs 1963). Convective diffusion coefficients can be expected to decrease approaching the wall until they are damped out at the laminar boundary layer making this solution irrelevant for a static chamber. However when the drum is rotated and convective diffusion is reduced sufficiently by buoyant air gyration and rapid viscous dissipation of small eddies, then gravitational gyration of aerosol particles can clear a zone extending one radius of gyration away from the wall. Because this clear zone extends well beyond the laminar boundary layer we can anticipate the possibility that convective diffusion might dominate transport by centrifugal drift into the clear zone where gravitational gyration becomes the dominant transport mechanism. This can be incorporated into the diffusion solution simply by reducing chamber radius by the radius of gyration

$$\frac{N}{N_0}=\frac{6}{\pi^2}\sum_{j=1}^{\infty}\frac{\exp\left(D\pi^2 j^2 t/(R-r_0)^2\right)}{j^2} \tag{1.9}$$

calculations using this equation appear in Figure 51 for the largest and smallest radii of gyration corresponding respectively to d = 6 μm at 1 rpm and d = 1 μm at 8 rpm for convective diffusion coefficients of $10^{-2}, 10^{-3}$ *and* 10^{-4} cm^2/s giving concentrations roughly in the range measured. These six curves show little dependence on particle size and rotation rate resulting in essentially three curves defined by the three diffusion coefficients which are intrinsically independent of particle size over the range of interest (Fuchs 1964) and probably independent of rotation rate. Therefore, the observed particle size and rpm dependence in concentration decay rate is not explained by the diffusion model. These convective diffusion coefficients are smaller than those encountered in the stirred settling model representing the rpm = 0 case where diffusion maintains essentially homogeneous concentration in spite of gravitational settling.

When transport into the clear zone by diffusion and drift due to centrifugal force are comparable in magnitude, we look to the steady state one dimensional solution (Friedlander 1977) for insight. The deposition flux at the clear zone boundary, where $n = n_b$, is

$$J=\frac{v_c n_b}{1-\exp[-v_c(R-r_0)/D]} \tag{1.10}$$

the ratio $J(D=0)/J$ of deposition flux due to centrifugal drift alone over that due to combined diffusion and drift is plotted in Figures 52 and 53 for convective diffusion coefficients of 0.1 *and* 0.01 cm^2/s over a range of particle sizes and rates of rotation. It is evident that diffusion which is intrinsically independent of particle size will have the greatest influence on progressively smaller particle transport as the rate of rotation increases such that $v_c(R-r_0)/D \le 1$. While convective diffusion significantly speeds up the concentration decay rate of smaller particles at lower rates of rotation and will help predict the measurements for

these particles, it has a negligible effect on the larger particles especially at the higher rates of rotation and will not help reconcile the large differences between Moss's theory predictions and the measurements.

Another pertinent result for combined drift plus diffusion is found in the transient solution for aerosol transport through an electrostatic precipitator (Shapiro 1998) in the limit where the Peclet number, drift velocity multiplied by transport distance divided by turbulent diffusion coefficient is greater than 10. Here diffusion slows the deposition process according to the equation

$$\frac{n}{n_0} = \exp\left(-\frac{v^2 t}{4D}\right) \tag{1.11}$$

and works to decrease rather than increase the aerosol concentration decay rate further increasing differences between measurements and Moss's theory predictions.

Although aerosol particle number concentrations were on the order of $\sim 700/cm^3$ during the drum experiments, the long residence times in the chamber could allow coagulation to play a role along with centrifugal drift in reducing aerosol concentration. To develop this approach we note that homogeneous concentration is maintained outside the clear zone in the rotating drum regardless of convective diffusion if initial concentration is homogeneous. This is because the rate of change of aerosol concentration, equal to the rate of change of aerosol mass in a volume element (flux in minus flux out) divided by that volume element, is independent of radius for centrifugal acceleration which is proportional to radius. The total number of aerosol particles in the rotating drum can be written

$$N = \pi\left(R - r_0\right)^2 L\, n \tag{1.12}$$

where L *and* n are the drum length and homogeneous aerosol number concentration. The rate of change in total number is equal to the flux of particles entering the clear zone

$$\frac{dN}{dt} = -2\pi\left(R - r_0\right) L v_c n \tag{1.13}$$

where $v_c = \omega^2\left(R - r_0\right)\tau$ is the particle velocity due to centrifugal force. Combining the two previous equations we get

$$-\frac{dn}{dt} = \frac{2v_c}{\left(R - r_0\right)} n \tag{1.14}$$

or simply

$$-\frac{dn}{dt} = 2\omega^2 \tau n \tag{1.15}$$

The combined effects of loss due to centrifugal drift which is proportional to the number concentration and coagulation which is proportional to the number concentration squared are represented by the differential equation

$$-\frac{dn}{dt} = \beta n + K n^2 \tag{1.16}$$

for a homogeneous cloud which has the solution (Fuchs 1964)

$$n = \frac{1}{\left(\frac{1}{n_0} + \frac{K}{\beta}\right)\exp(\beta t) - \frac{K}{\beta}} \tag{1.17}$$

where n_0 *and* K are the initial concentration and coagulation constant with $\beta = 2\omega^2\tau$. In the limit where coagulation becomes negligible, $K \to 0$ this expression provides the solution for aerosol concentration as a function of time when loss is due to centrifugal drift into the clear zone. Over the particle size range of interest Brownian coagulation sets a lower limit of $\sim 10^{-9}\, cm^3/s$ and differential gravitational settling (in our case gyration) sets an upper limit of $\sim 10^{-8}\, cm^3/s$ for the coagulation constant or more precisely for the collision frequency function (Friedlander 1977). The ratio of the number concentration remaining after 48 hours with over without coagulation is surface plotted in Figures 54 and 55 as a function of particle size and rpm for the lower and upper limit values of the coagulation constant. The influence of coagulation can be seen to be significant in reducing number concentration of smaller particles at all rates of rotation and larger particles at the slower rates of rotation but has negligible effect on the number concentration of larger particles at higher rotation rates. Therefore, differences between measurements and Moss's theory predictions for larger particles at higher rotation rates are not explained.

During these experiments the SRI nozzle disseminated aluminum oxide powder by aspirating the powder into and through an aluminum tube using a sonic velocity air flow surrounding the tube outlet and moving along the tube axis away from the tube. This same airflow then projects the powder out in a jet that dilutes the aerosol concentration while inducing turbulence that deagglomerates aerosol particles. The aluminum tube is grounded and it is likely that triboelectric charging of the powder occurred as the powder moved through the tube and scraped the tube wall. A monopolar charged cloud produced in this way would expand because of mutual repulsion and aerosol concentration would decay due to a combination of centrifugal and electrostatic drift into the zone cleared by gravitational gyration. The component of aerosol particle velocity at the edge of the clear zone due to electrostatic repulsion between that particle and the monopolar charged cloud is

$$v_e = \frac{\tau F_e}{m} \tag{1.18}$$

where F_e/m is the acceleration due to the electrostatic force, F_e, acting on an aerosol particle of mass m. The electrostatic force is

$$F_e = \frac{qQ}{(R - r_0)^2} \tag{1.19}$$

where q is the charge on the particle and Q is the total charge of a spherical monopolar charged cloud. If we assume that the amount of triboelectric charge is proportional to the surface area of the particle, then

$$q = \xi e d^2 \tag{1.20}$$

where ξd^2 is the number of elementary charges $e = 4.8 X 10^{-10} esu$ on the particle in the cgs unit system. Assuming homogeneous concentration, the total charge contained in the cloud is

$$Q = \frac{4}{3}\pi (R - r_0)^3 n\xi e \langle d^2 \rangle \tag{1.21}$$

where $\langle d^2 \rangle$ reflects an average surface area per particle which depends on the instantaneous size distribution. The diameter corresponding to $\sqrt{\langle d^2 \rangle}$ is called the diameter of the average surface and can be related to the more common number median diameter

$$d_{nmd} = \sqrt{\langle d^2 \rangle} \exp\left(-\ln^2 \sigma_g\right) , \tag{1.22}$$

surface median diameter

$$d_{smd} = \sqrt{\langle d^2 \rangle} \exp\left(\ln^2 \sigma_g\right) \tag{1.23}$$

and mass median diameter

$$d_{mmd} = \sqrt{\langle d^2 \rangle} \exp\left(2 \ln^2 \sigma_g\right) \tag{1.24}$$

descriptors (Reist 1993) for a log normal size distribution where σ_g is the geometric standard deviation.

The equation controlling aerosol concentration when coagulation, drift due to centripetal acceleration and drift due to triboelectric charging occur simultaneously in a homogeneous cloud becomes

$$-\frac{dn}{dt}=\beta n+Kn^2+\frac{2v_e}{\left(R-r_0\right)}n \tag{1.25}$$

which can be written

$$-\frac{dn}{dt}=\beta n+K^{'}n^2 \tag{1.26}$$

with the solution

$$n=\frac{1}{\left(\frac{1}{n_0}+\frac{K^{'}}{\beta}\right)\exp(\beta t)-\frac{K^{'}}{\beta}} \tag{1.27}$$

where

$$K^{'}=K+\frac{\beta_e}{n} \tag{1.28}$$

and

$$\beta_e=\frac{2v_e}{\left(R-r_0\right)}=\frac{8\pi\tau\xi^2 d^2\left\langle d^2\right\rangle e^2 n}{3m} \tag{1.29}$$

Figure 56 shows the ratio of aerosol remaining after 24 hours with triboelectric charging of $10d^2$ electron charges per particle (d in microns) over that remaining without triboelectric charging. Initial concentration is 700 particles per cubic centimeter, $\left\langle d^2\right\rangle=9$ microns and coagulation is ignored. The magnitude of β_e/n for this case is plotted in Figure 57 and exceeds K over almost the entire size distribution range. Figures 58 through 63 show the combined effects of centrifugal and electrostatic drift at this charging level into the clear zone along with coagulation at the Brownian level $K=10^{-9}\ cm^3/s$ assuming the coulombic repulsive force reduces coagulation to that level. If the van der Waals forces holding the particles together after they make contact do not exceed the repulsive force then the coagulation constant would have to be set to zero but it has little effect on the results at the Brownian level. Because $\left\langle d^2\right\rangle$ continually decreases as the charged cloud decays, we found that by simply setting it equal to d^2 the measurements could be matched better than by using any fixed value. Thus the smaller particles which are suspended longer on average encounter a repulsive cloud of particles minus the larger particles. The larger particles suspended for shorter times see a cloud of all particle

sizes but the larger particles have much greater charge per particle and represent the majority of charge existing in the cloud. Predictions using this approach are in reasonable agreement with the measurements and a closer fit would no doubt be possible by further adjusting the charging coefficient ξ or $\langle d^2 \rangle$.

5. CONCLUSIONS

The stirred settling model was found suitable for the drum chamber when it is not rotated. Small temperature differences between the wall and aerosol drive convective diffusion that is sufficient to uniformly mix particle up to 10 μm aerodynamic diameter even in a one meter high chamber. Once the drum is rotated at rates of 1 to 8 rpm convective diffusion is damped out by convective flow gyration and viscous dissipation of small eddies. Transport is then governed by centrifugal drift into a clear zone as described by Gruel's and Moss's theories. This zone extends one radius of gyration away from the wall is rapidly cleared by gravitational gyration in one revolution. During the experiment electrostatic forces due to triboelectric charging within our dissemination device were found to be important in determining the aerosol decay rate at the rates of rotation studied. Triboelectric charging in the dissemination device used has always led to negligible effects during static aerosol chamber tests. This is because the electrical forces involved are small compared to gravity in controlling transport through the laminar boundary layer and small compared to convective or turbulent diffusion controlling transport outside the laminar boundary layer. The rotating drum essentially replaces gravity with centrifugal force and at the same time damps out convective diffusion. Instead of gravitational settling through the laminar boundary layer the rotating drum has gravitational gyration through the laminar boundary layer and a clear zone extending one radius of gyration away from the wall that is created in one revolution. In order to study other phenomena involving forces that are small compared to gravity, triboelectric charging should be avoided by neutralizing the aerosol as it exits the dissemination device or the dissemination method should be altered to avoid triboelectric charging.

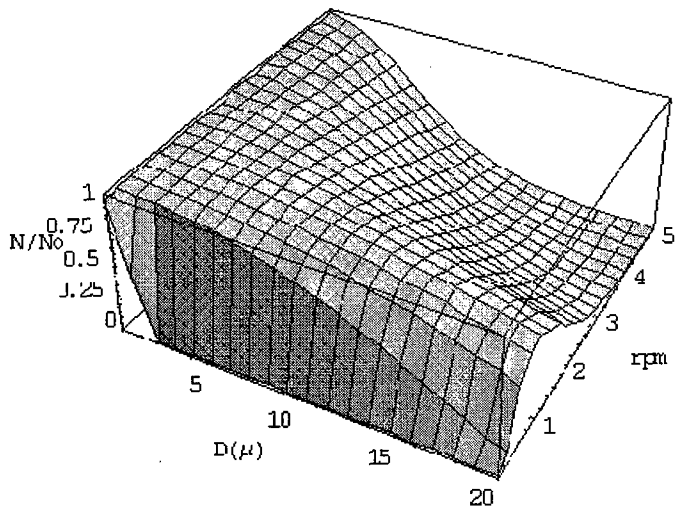

Figure 1. Aerosol Rotating Drum (R = 50 cm, t = 3 hr)

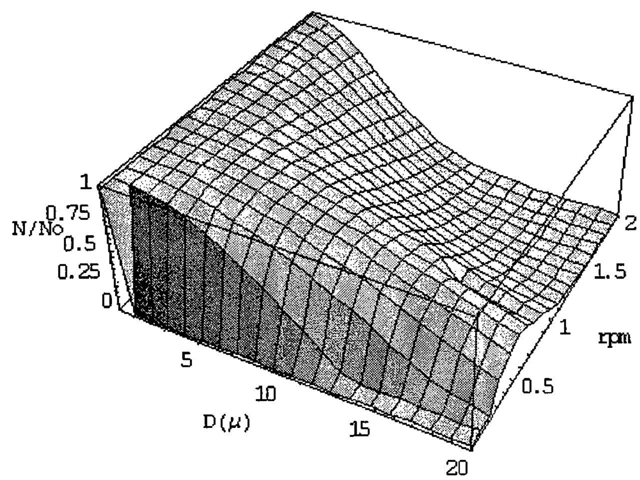

Figure 2. Aerosol Rotating Drum (R = 50 cm, t = 24 hr)

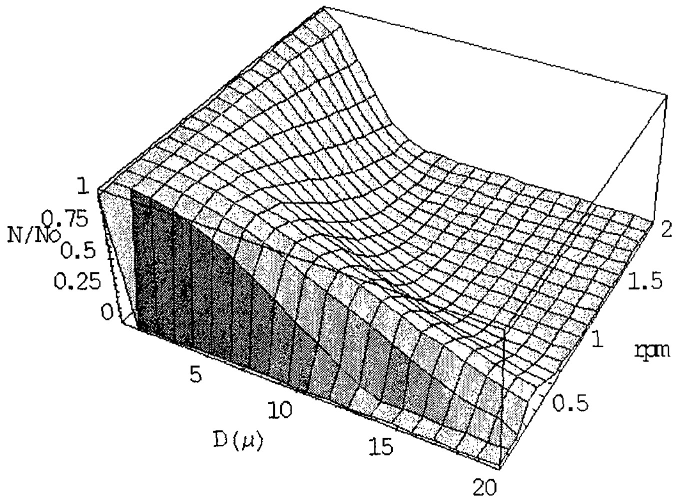

Figure 3. Aerosol Rotating Drum (R = 50 cm, t = 96 hr)

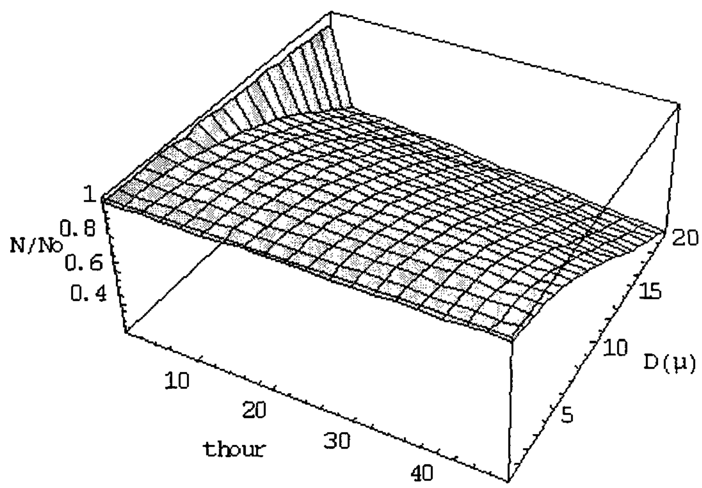

Figure 4. Aerosol Rotating Drum (R = 50cm, rpm = .5)

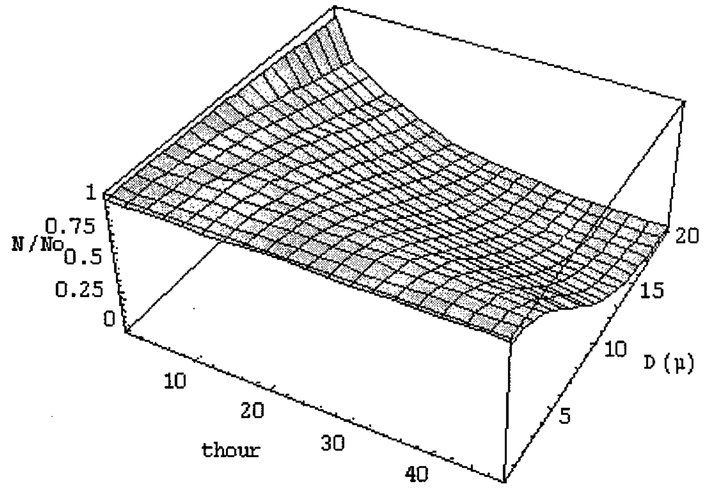

Figure 5. Aerosol Rotating Drum (R = 50 cm, rpm = 1)

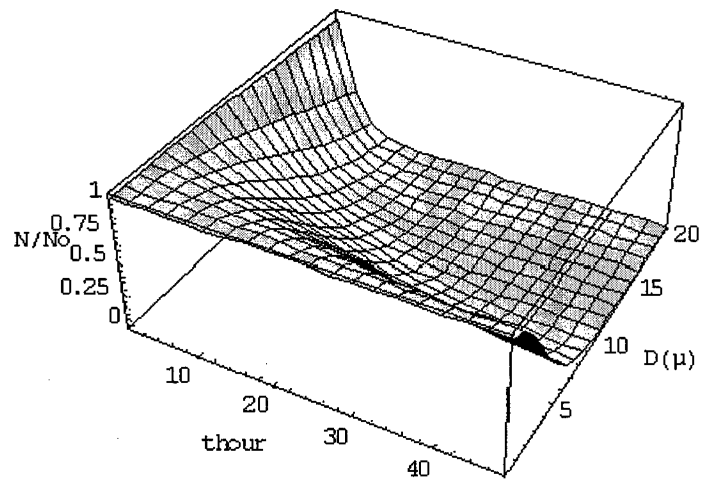

Figure 6. Aerosol Rotating Drum (R = 50 cm, rpm = 2)

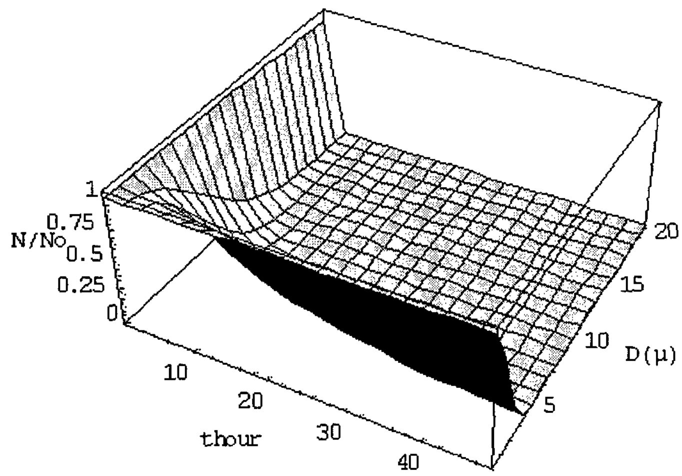

Figure 7. Aerosol Rotating Drum (R = 50 cm, rpm = 5)

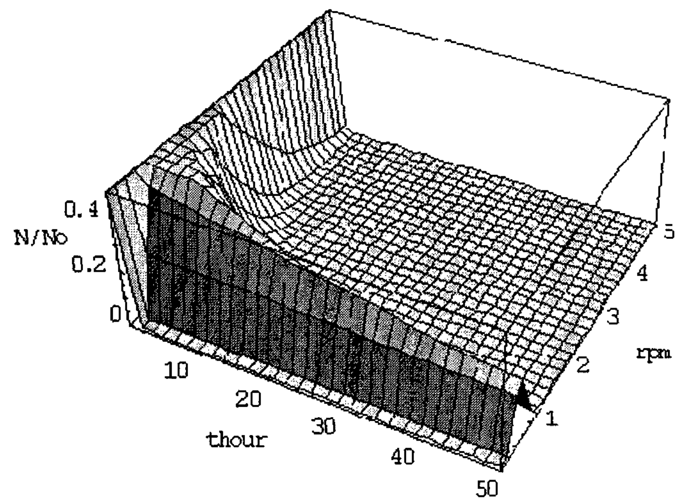

Figure 8. Aerosol Rotating Drum (R = 50 cm, D = 20μ)

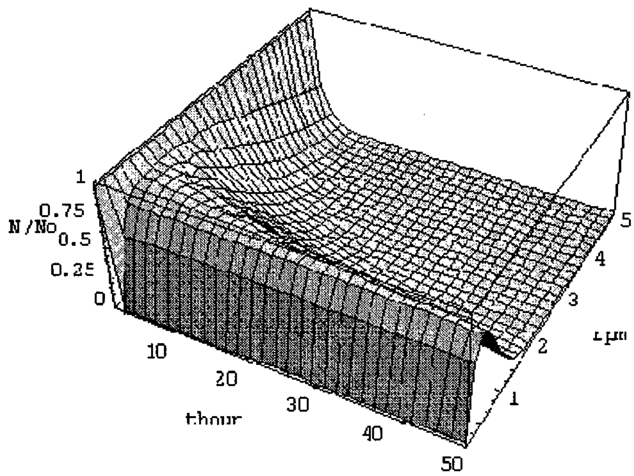

Figure 9. Aerosol Rotating Drum (R = 50 cm, D = 10μ)

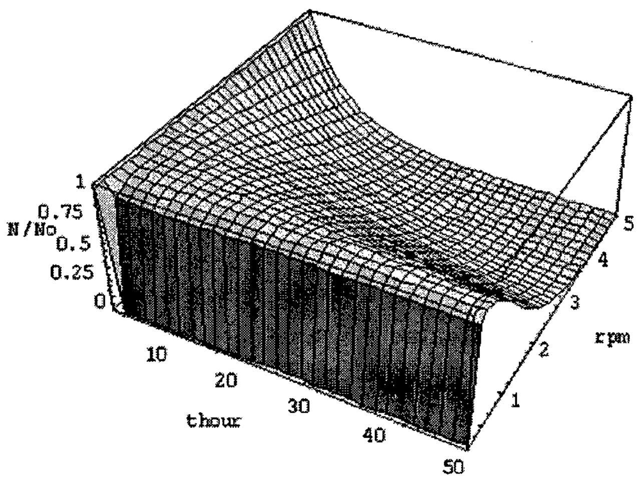

Figure 10. Aerosol Rotating Drum (R = 50 cm, D = 5μ)

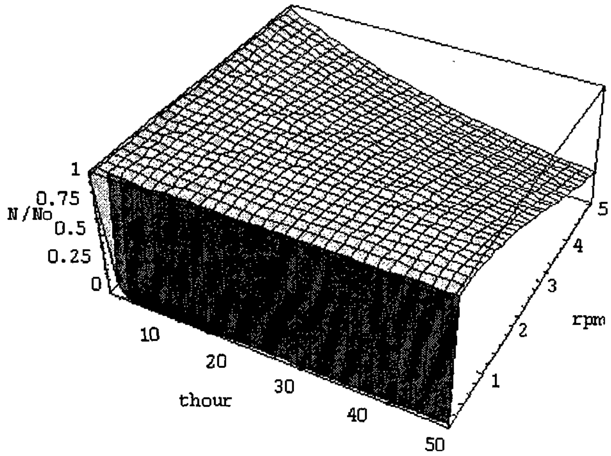

Figure 11. Aerosol Rotating Drum (R = 50 cm, D = 2μ)

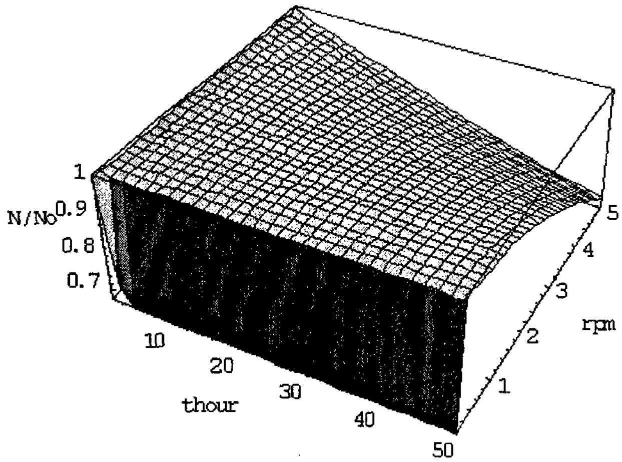

Figure 12. Aerosol Rotating Drum (R = 50 cm, D = 1μ)

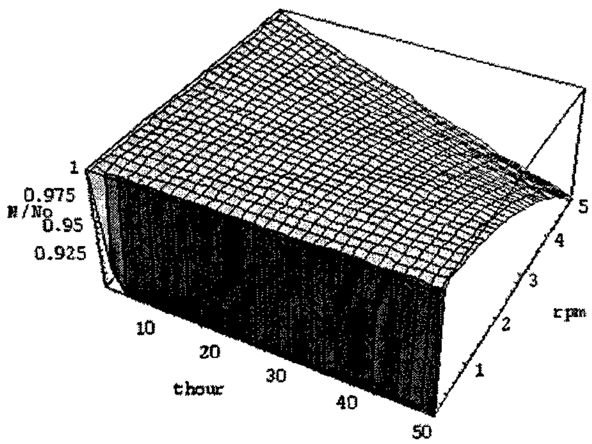

Figure 13. Aerosol Rotating Drum (R = 50 cm, D = .5μ)

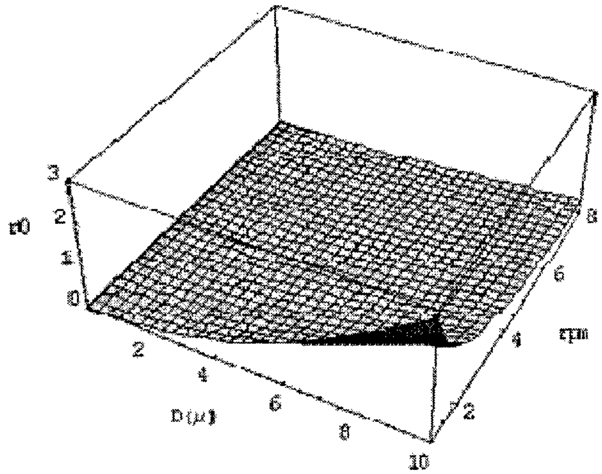

Figure 14. Radius of Gyration r0 (cm)

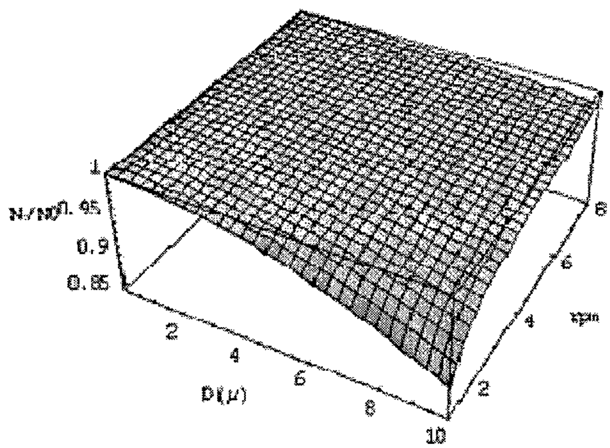

Figure 15. N/N0 after One Revolution; R = 50 cm

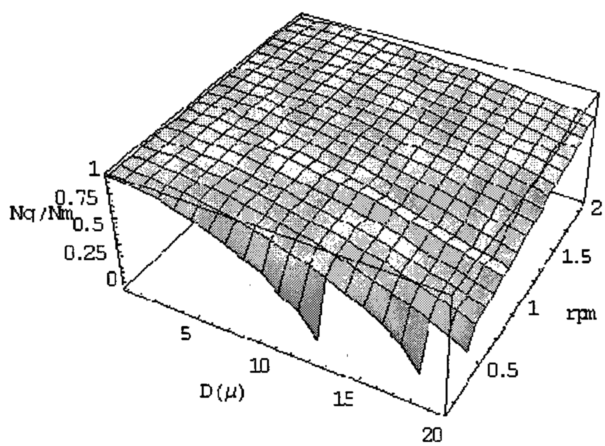

Figure 16. Aerosol Rotating Drum (R = 50 cm, t = 3 hr)

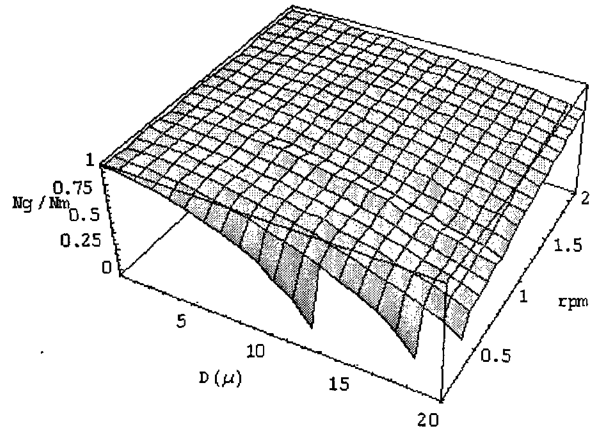

Figure 17. Aerosol Rotating Drum (R = 50 cm, t = 24 hr)

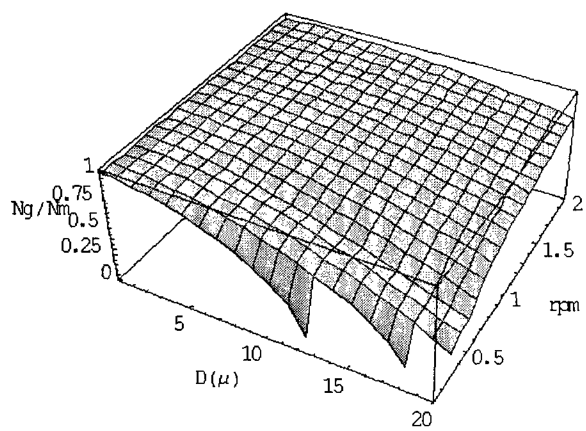

Figure 18. Aerosol Rotating Drum (R = 50 cm, t = 96 hr)

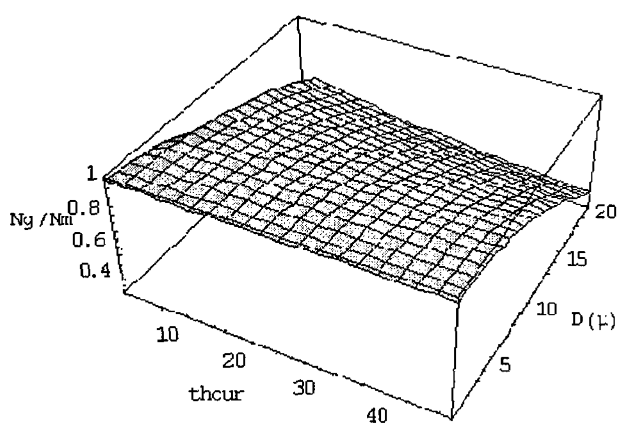

Figure 19. Aerosol Rotating Drum (R = 50 cm, rpm = .5)

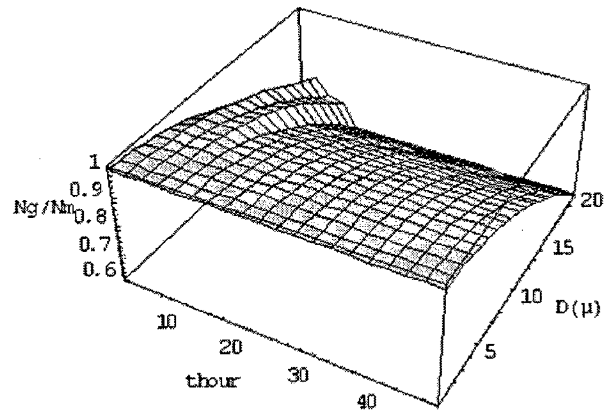

Figure 20. Aerosol Rotating Drum (R = 50 cm, rpm = 1)

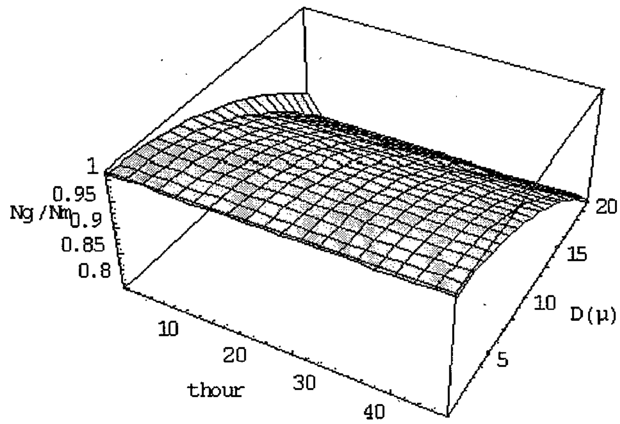

Figure 21. Aerosol Rotating Drum (R = 50 cm, rpm = 2)

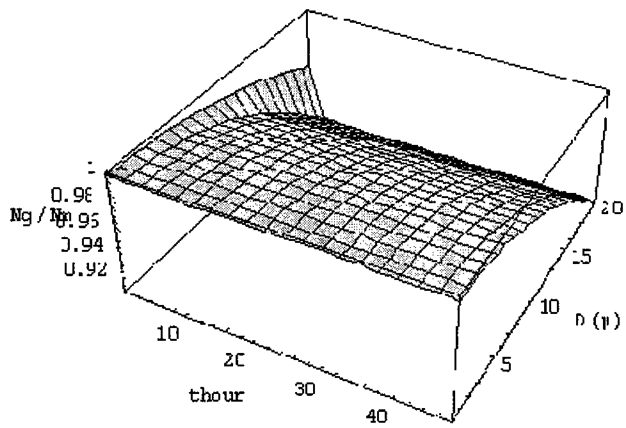

Figure 22. Aerosol Rotating Drum (R = 50 cm, rpm = 5)

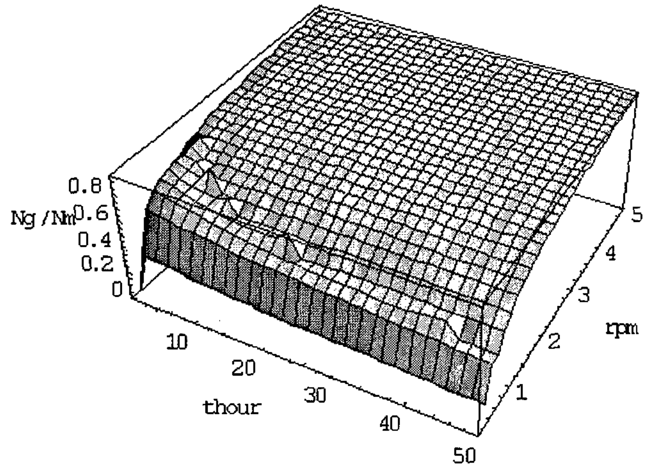

Figure 23. Aerosol Rotating Drum (R = 50 cm, D = 20μ)

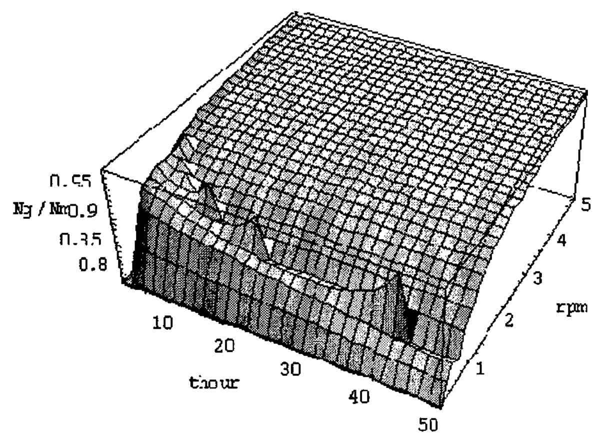

Figure 24. Aerosol Rotating Drum (R = 50 cm, D = 10μ)

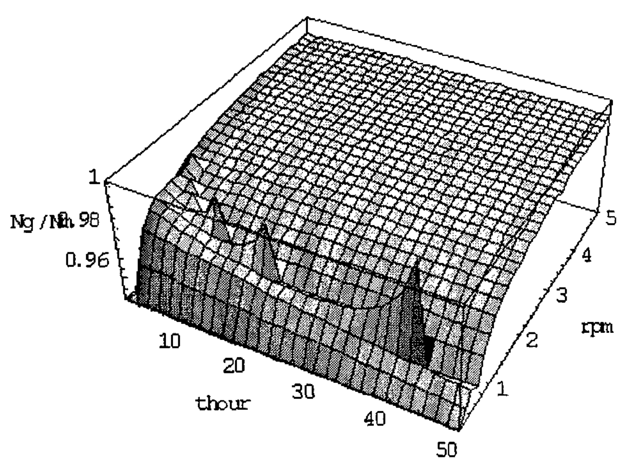

Figure 25. Aerosol Rotating Drum (R = 50 cm, D = 5μ)

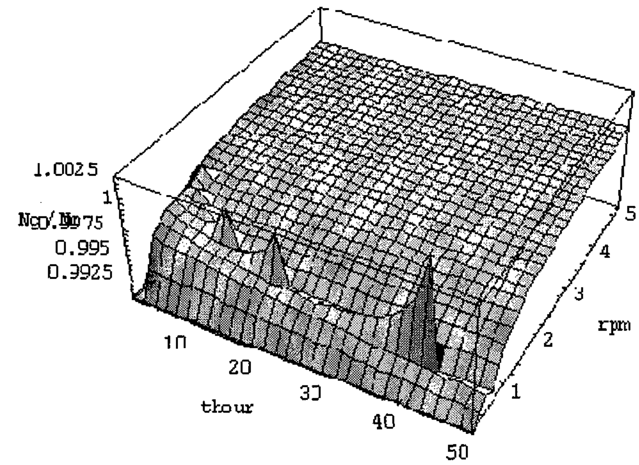

Figure 26. Aerosol Rotating Drum (R = 50 cm, D = 2μ)

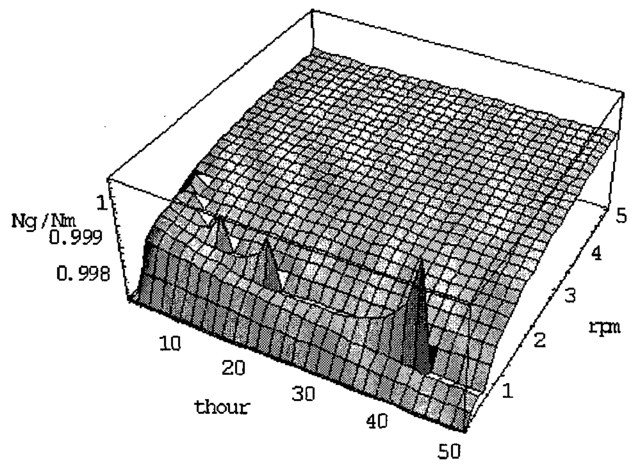

Figure 27. Aerosol Rotating Drum (R = 50 cm, D = 1μ)

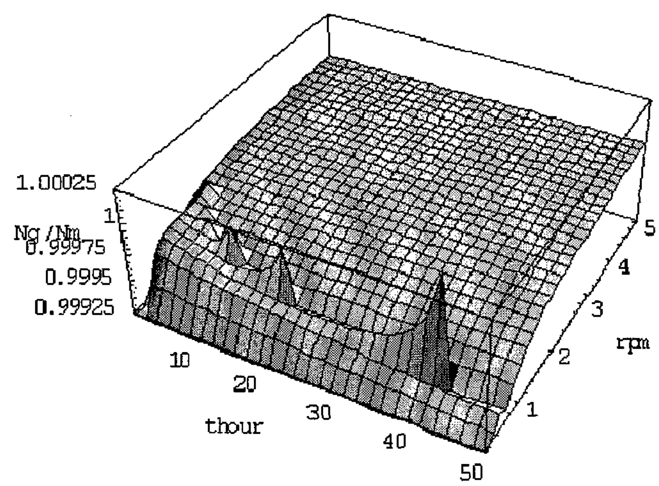

Figure 28. Aerosol Rotating Drum (R = 50 cm, D = .5μ)

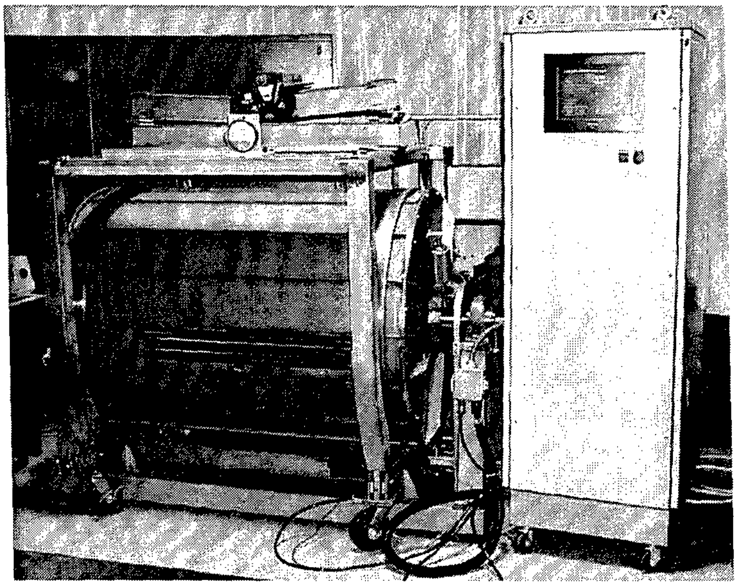

Figure 29. Rotating Drum Used for this Study and Borrowed from Owen Moss

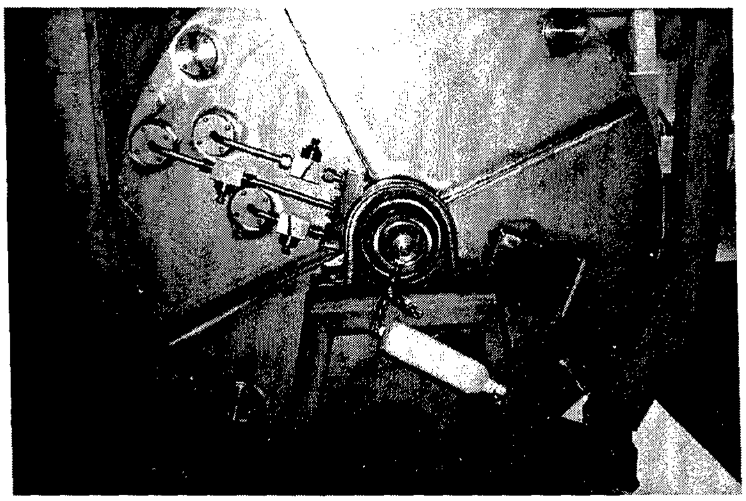

Figure 30. Four Sampling Ports on Drum Wall Opposite Piston Wall

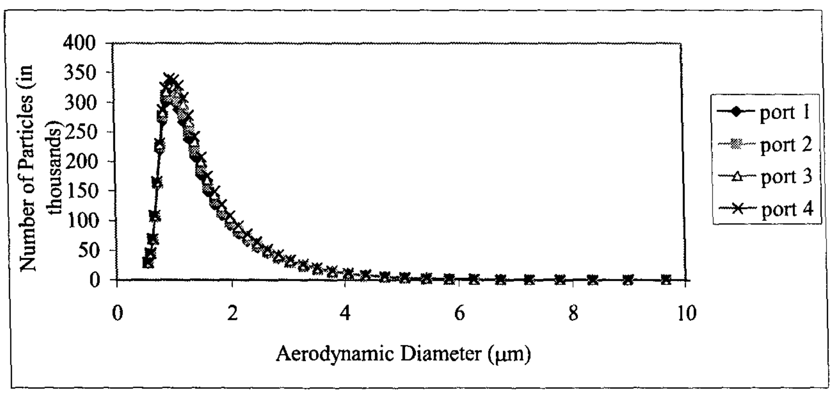

Figure 31. Initial Aerosol Concentration and Size Distribution Measured in the Drum

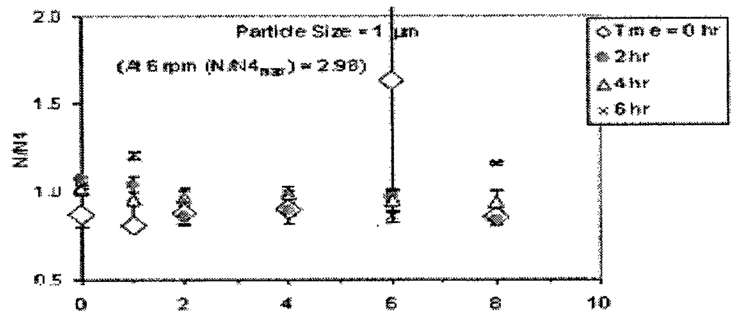

Figure 32. Ratio of Measured Concentration of 1 μm Particles in Ports 1, 2, and 3 Over that in Port 4 (Axis Port) at Various Times and as a Function of RPM Rotation Rate Plotted on the Abcissa.

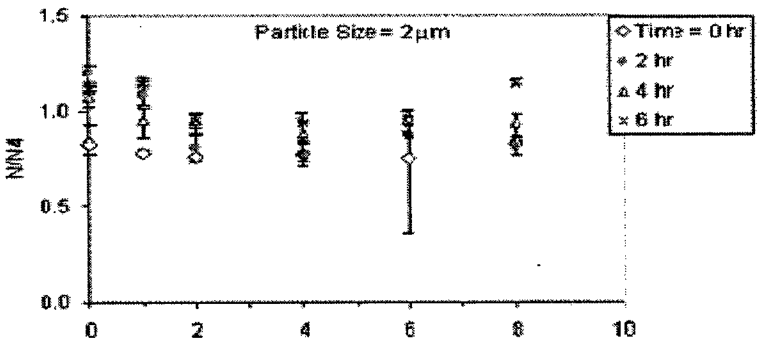

Figure 33. Ratio of Measured Concentration of 2 μm Particles in Ports 1, 2, and 3 Over that in Port 4 (Axis Port) at Various Times and as a Function of RPM Rotation Rate Plotted on the Abcissa.

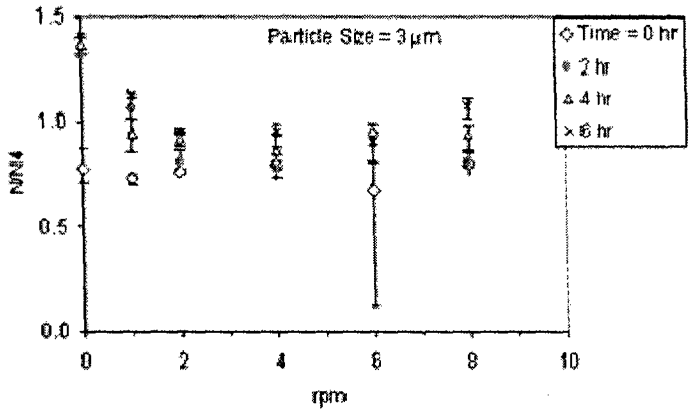

Figure 34. Ratio of Measured Concentration of 3 µm Particles in Ports 1, 2, and 3 Over that in Port 4 (Axis Port) at Various Times and as a Function of RPM Rotation Rate Plotted on the Abcissa.

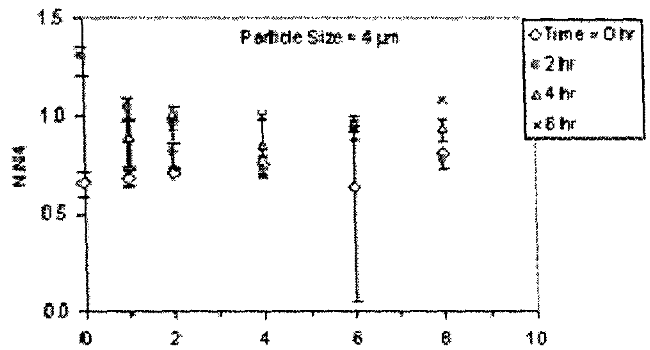

Figure 35. Ratio of Measured Concentration of 4 µm Particles in Ports 1, 2, and 3 Over that in Port 4 (Axis Port) at Various Times and as a Function of RPM Rotation Rate Plotted on the Abcissa.

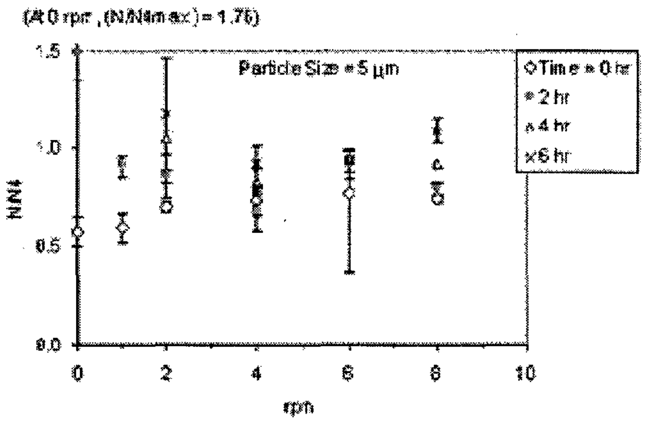

Figure 36. Ratio of Measured Concentration of 5 µm Particles in Ports 1, 2, and 3 Over that in Port 4 (Axis Port) at Various Times and as a Function of RPM Rotation Rate Plotted on the Abcissa.

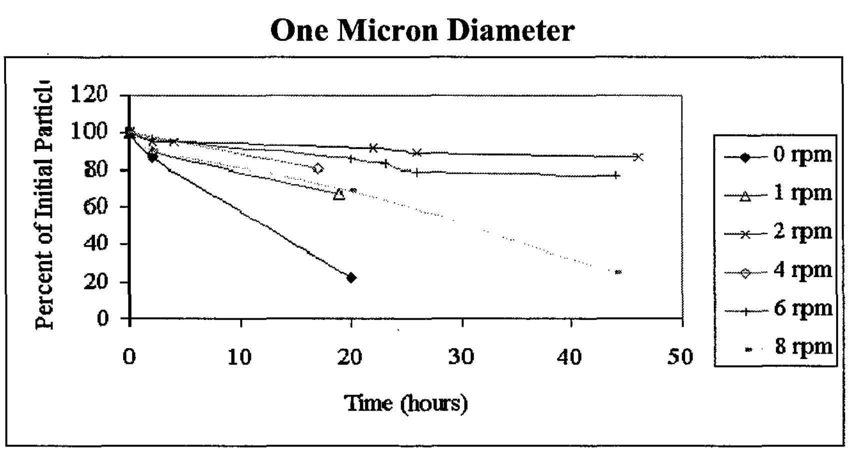

Figure 37. Concentration of 1 µm Diameter Particles Measured through Axis Port

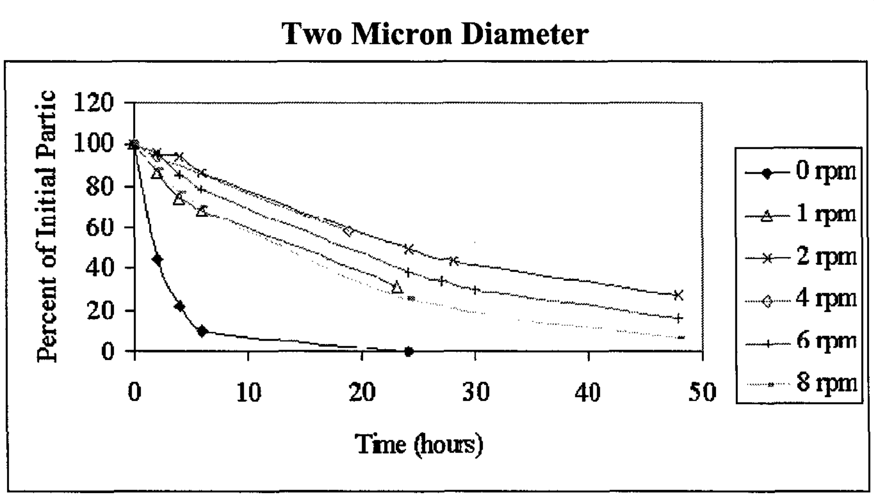

Figure 38. Concentration of 2 µm Diameter Particles Measured through Axis Port

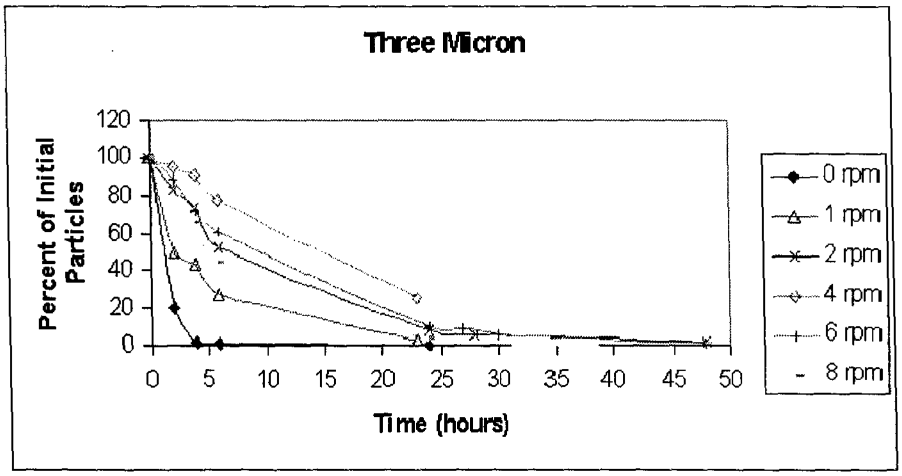

Figure 39. Concentration of 3 μm Diameter Particles Measured through Axis Port

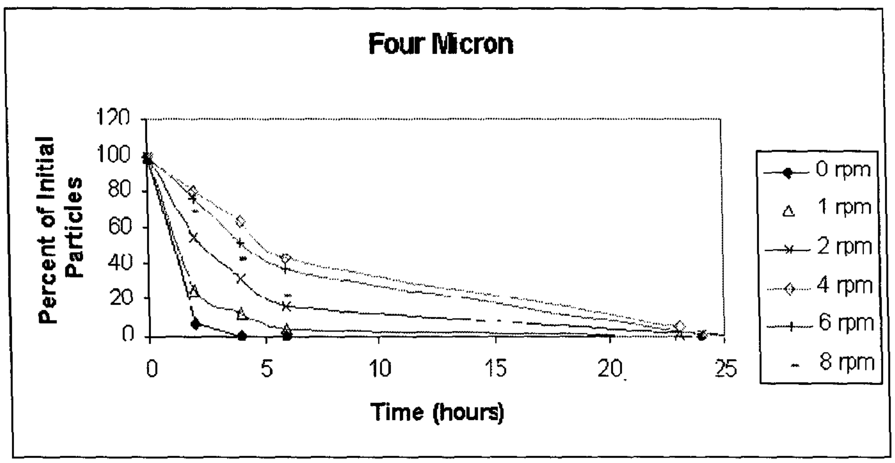

Figure 40. Concentration of 4 μm Diameter Particles Measured through Axis Port

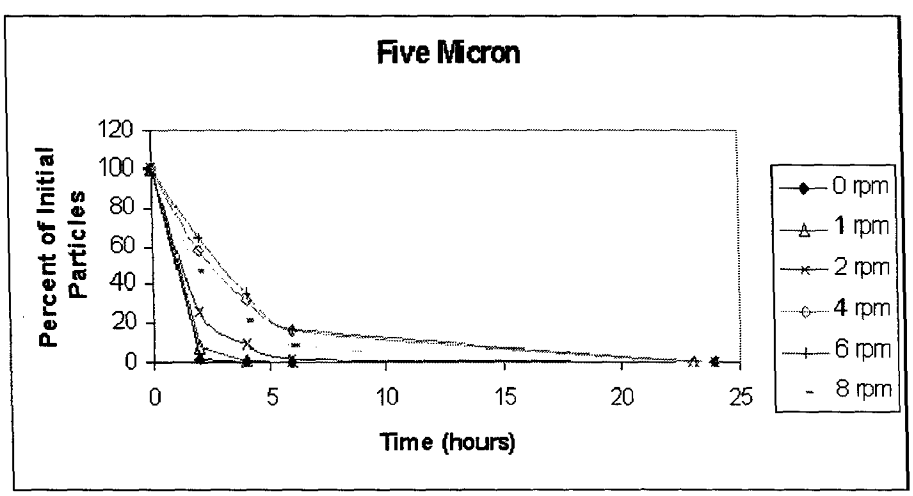

Figure 41. Concentration of 5 μm Diameter Particles Measured through Axis Port

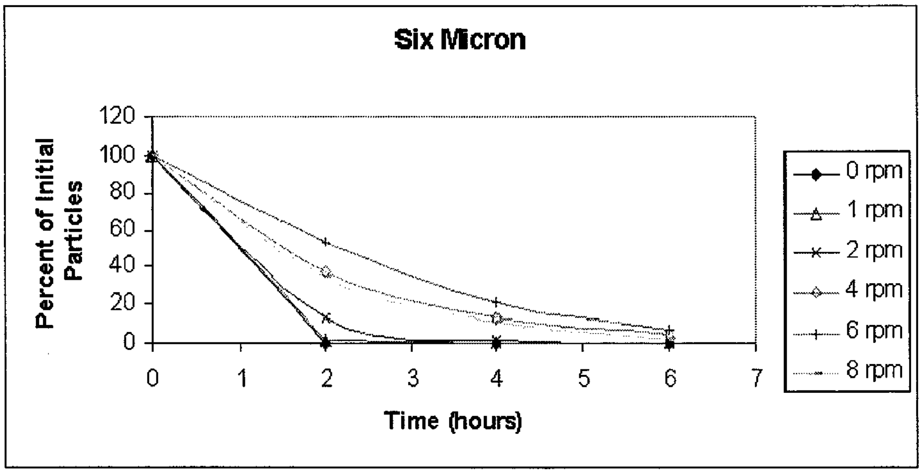

Figure 42. Concentration of 6 μm Diameter Particles Measured through Axis Port

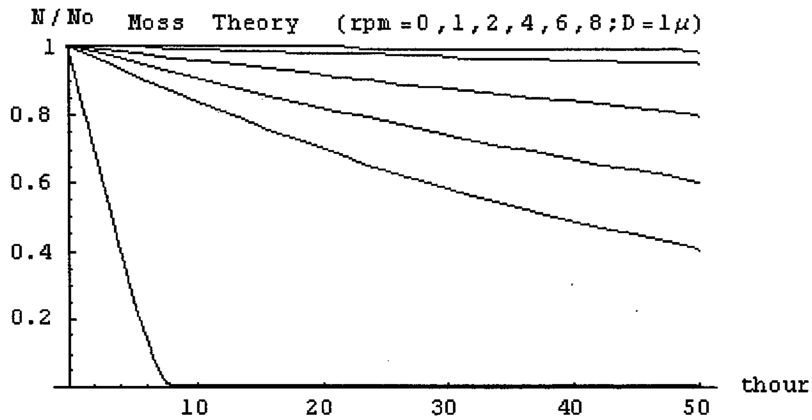

Figure 43. Asgharian and Moss Theory Predictions for 1 μm Particles

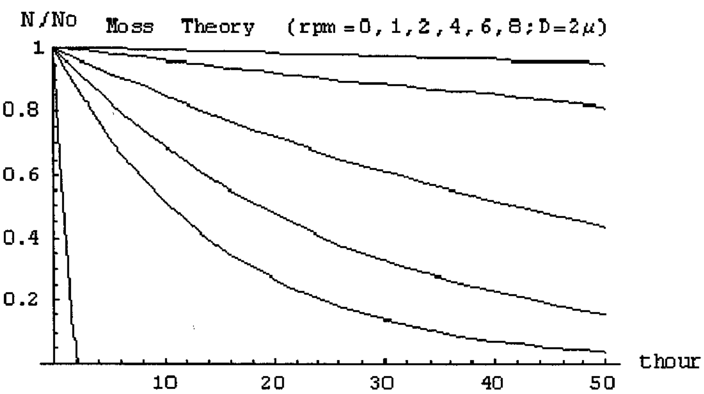

Figure 44. Asgharian and Moss Theory Predictions for 2 μm Particles

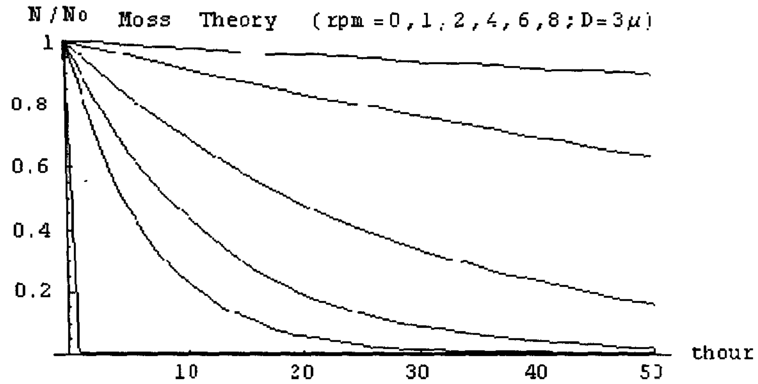

Figure 45. Asgharian and Moss Theory Predictions for 3 μm Particles

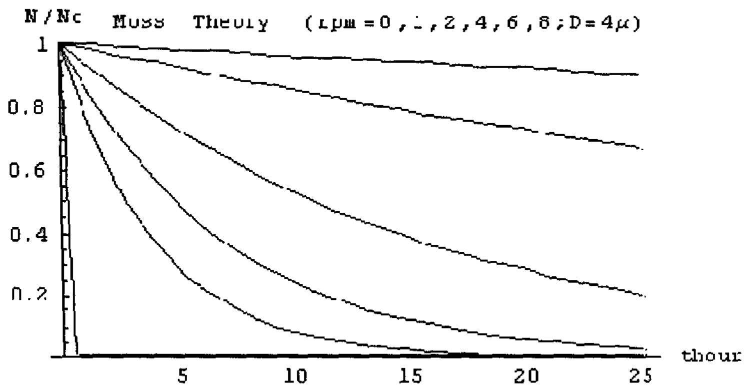

Figure 46. Asgharian and Moss Theory Predictions for 4 μm Particles

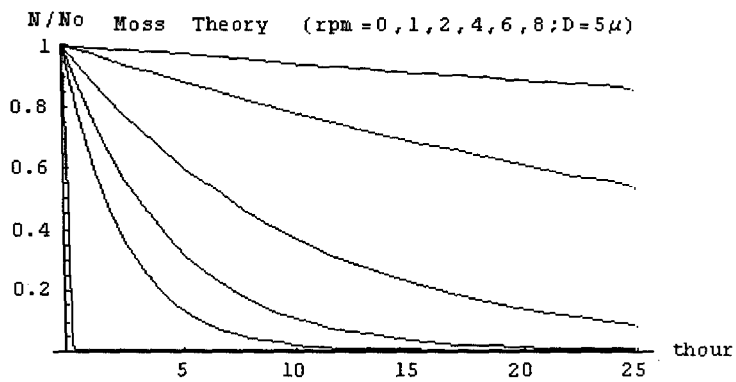

Figure 47. Asgharian and Moss Theory Predictions for 5 μm Particles

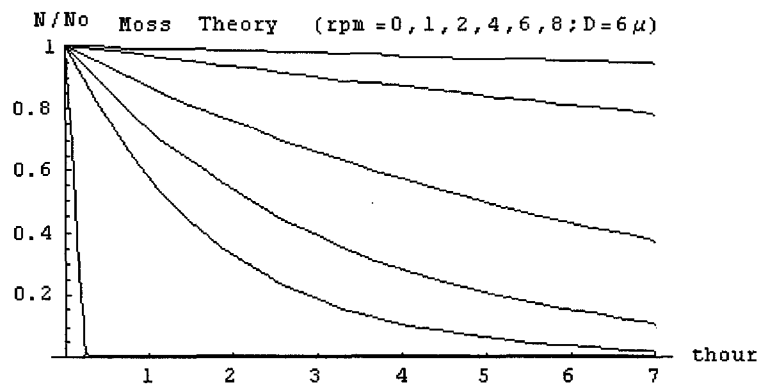

Figure 48. Asgharian and Moss Theory Predictions for 6 μm Particles

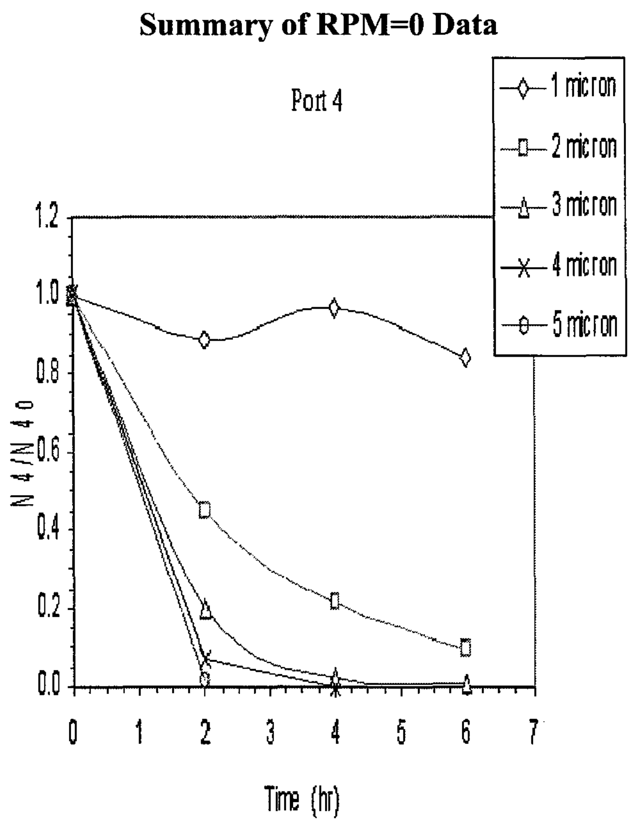

Figure 49. Summary of Static (rpm = 0) Drum Measurements through Axis Port

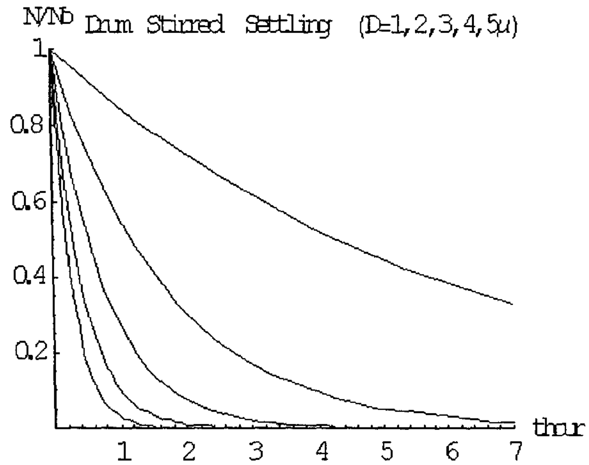

Figure 50. Stirred Settling Theory Predictions of 1, 2, 3, 4, and 5 μm Diameter Aerosol Concentration Fractions Remaining as a Function of Time in Hours

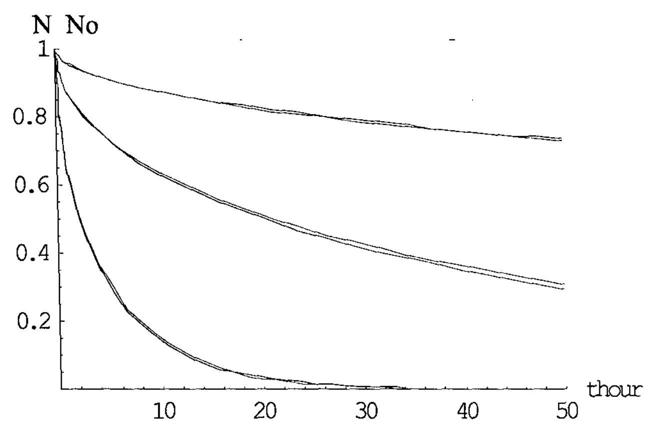

Figure 51. Diffusion Theory Predictions of Concentration Fraction Remaining as a Function of Time in Hours. Three Pairs of Curves Corresponding to Diffusion Coefficients of 0.01, 0.001, and 0.0001 cm^2/s at the Largest and Smallest Radii of Gyration

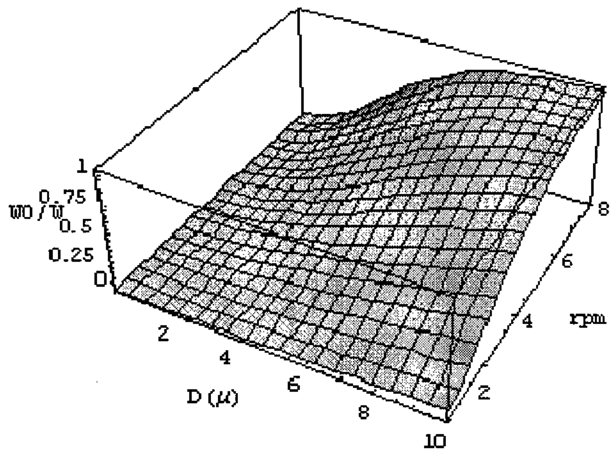

Figure 52. Deposition without/with Diffusion; D = .1cm^2/s

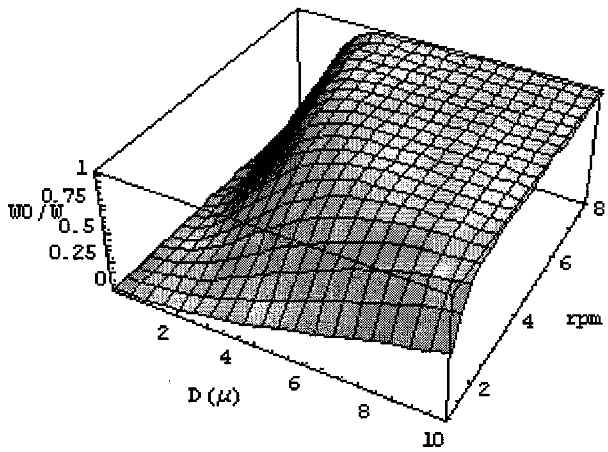

Figure 53. Deposition without/with Diffusion; D = .01cm^2/s

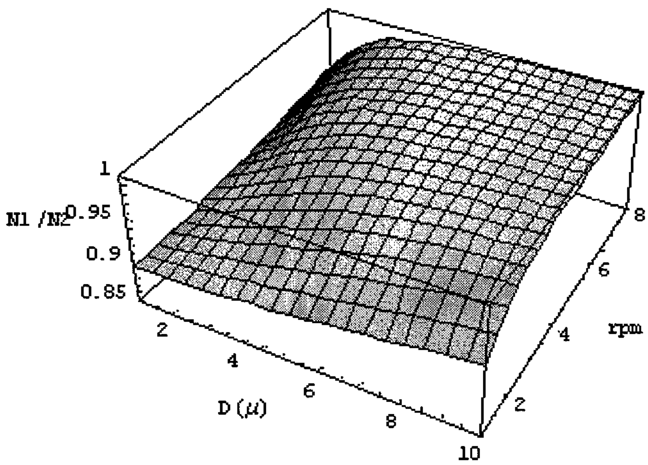

Figure 54. N1/N2 = N [K = 10^-9 /N (K = 0)]; No = 700, t = 48 hr

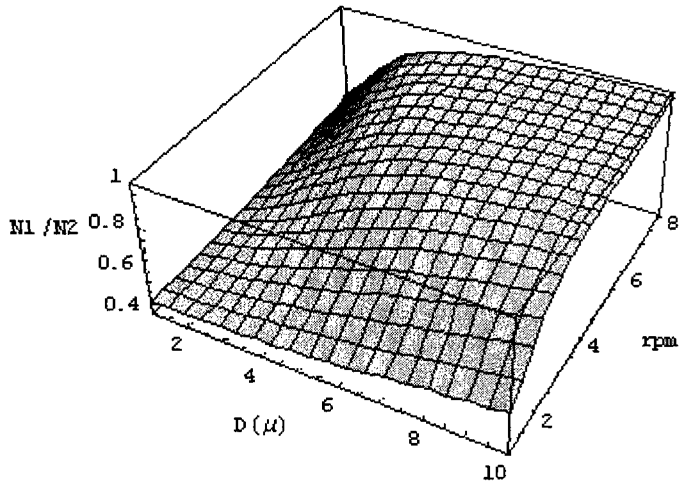

Figure 55. N1/N2 = N [K = 10^-8 /N (K = 0)]; No = 700, t = 48 hr

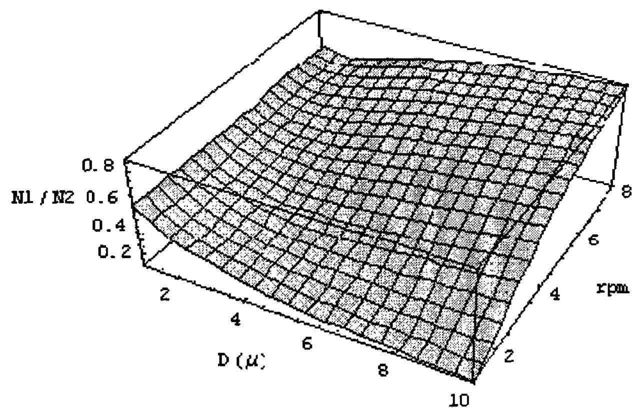

Figure 56. N1/N2 = N [K(Ncharge = 10∗D^2)/N (K = 0)]; No = 700, t = 24 hr

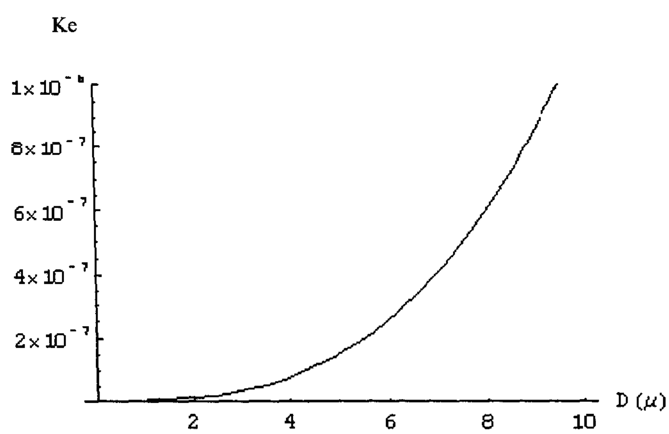

Figure 57. Electrostatic Dispersion (Ncharge = 10∗D^2)

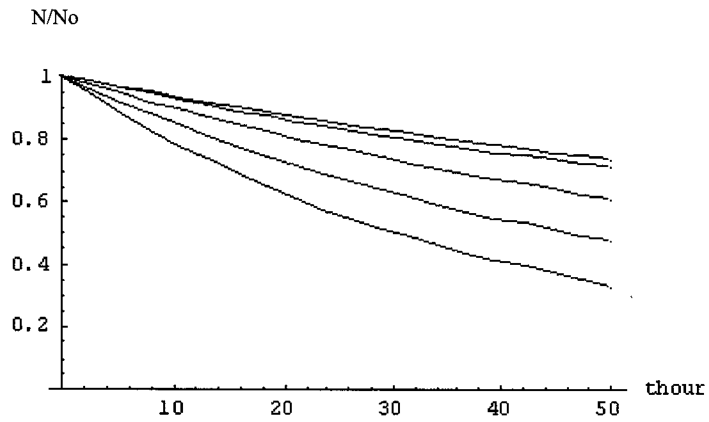

Figure 58. Ncharge = 10*D^2 (rpm = 1, 2, 4, 6, 8; D = 1μ)

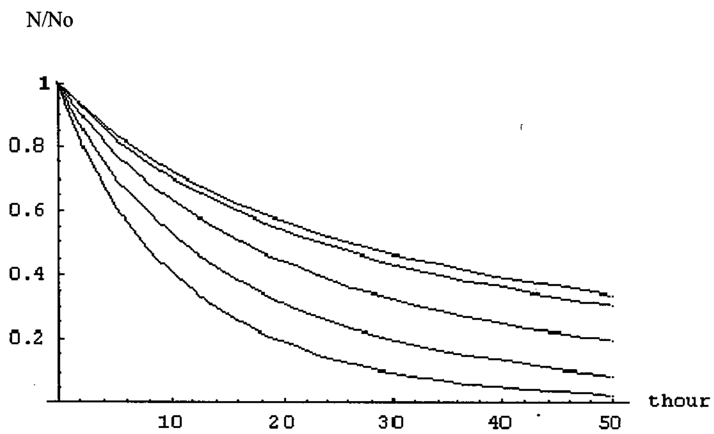

Figure 59. Ncharge = 10*D^2 (rpm = 1, 2, 4, 6, 8; D = 2μ)

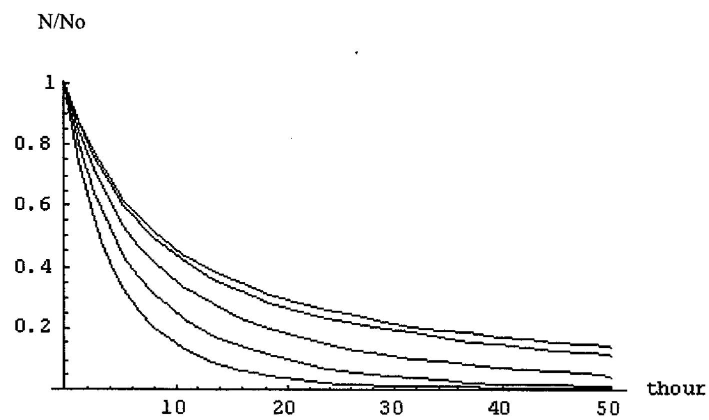

Figure 60. Ncharge = 10*D^2 (rpm = 1, 2, 4, 6, 8; D = 3μ)

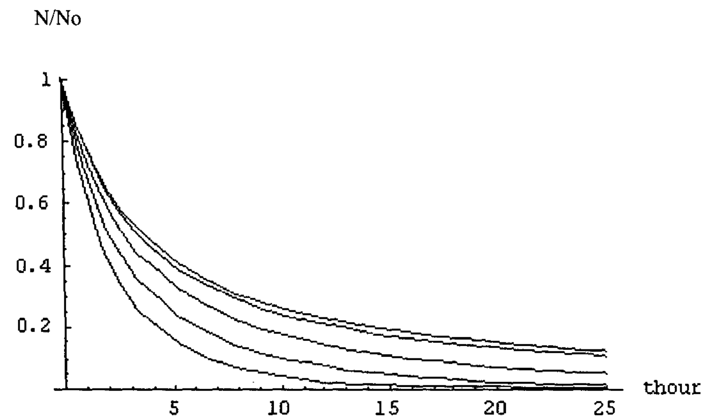

Figure 61. Ncharge = 10*D^2 (rpm = 1, 2, 4, 6, 8; D = 4μ)

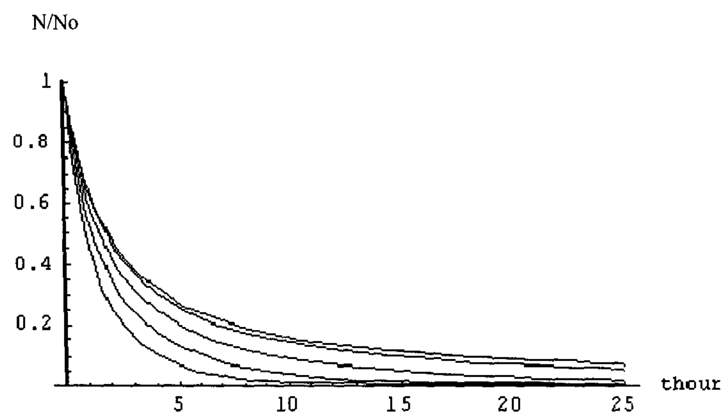

Figure 62. Ncharge = 10*D^2 (rpm = 1, 2, 4, 6, 8; D = 5μ)

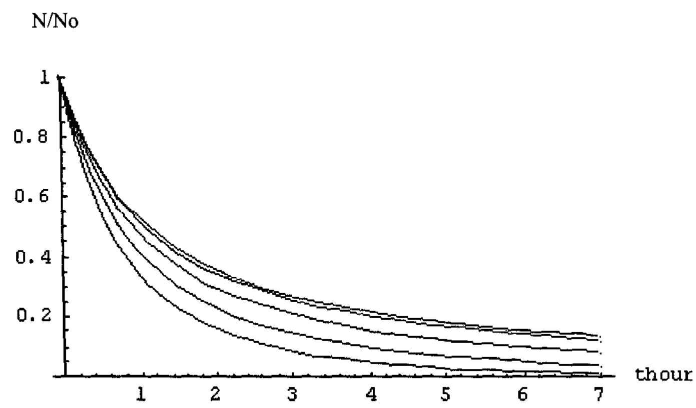

Figure 63. Ncharge = 10*D^2 (rpm = 1, 2, 4, 6, 8; D = 6μ)

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3. Sutton, T. *Decay of Particle Concentration as a Function of Rotation Rate in a Rotating Drum Chamber*, 2003 submitted for ECBC publication.

4. *The Mechanics of Aerosols*, Fuchs, N.A., Macmillan Company, New York, 1964.

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APPENDIX

MATHEMATICA CODE GENERATING SURFACE PLOTS

MATHEMATICA file MossN(D,RPM).nb

```
rho = 1;
mu = 1.83*10^-4;
d = DD*10^-4;
omega = 2*Pi*rpm/60;
t = thour*3600;
CC = 1 + (2*.071*10^-4/d)(1.257 + .4*Exp[-1.1*d/(2*.071*10^-4)]);
tau = rho*d^2 CC/(18*mu);
p = Exp[-2*tau*t*omega^2];
h = If[Pi <=
      omega*t, (tau*980/omega)(1 +
        p^.5), (tau*980/omega)(1 - 2*Cos[omega*t]*p^.5 + p)^.5];
h1 = (r*r + h*h - r*r*p)/(2*h);
h2 = h - h1;
r = 50;
f1 = p;
f2 = (1 + p)/2 -
    1/Pi*(h1/r*(1 - (h1/r)^2)^.5 + ArcSin[h1/r] + h2/r*(p - (h2/r)^2)^.5 +
        p*ArcSin[h2/r]);
f3 = 0;
hrlow = 1 - p^0.5;
hrhigh = 1 + p^0.5;
thour = 3;
g1 = Plot3D[
    If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {DD, 0.01,
      20}, {rpm, .0001, 5},
    PlotLabel -> "Aerosol Rotating Drum (R=50cm,t=3hr)",
    AxesLabel -> {"D(\[Mu])", "rpm", "N/No"}, PlotPoints -> 20];
thour = 24;
g1 = Plot3D[
    If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {DD, 0.01,
      20}, {rpm, .0001, 2},
    PlotLabel -> "Aerosol Rotating Drum (R=50cm,t=24hr)",
    AxesLabel -> {"D(\[Mu])", "rpm", "N/No"}, PlotPoints -> 20];
thour = 96;
g1 = Plot3D[
    If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {DD, 0.01,
      20}, {rpm, .0001, 2},
    PlotLabel -> "Aerosol Rotating Drum (R=50cm,t=96hr)",
    AxesLabel -> {"D(\[Mu])", "rpm", "N/No"}, PlotPoints -> 20];
```

```
g1 = Plot3D[h/r, {DD, 0.01, 20}, {rpm, .0001, 5},
    PlotLabel -> "Aerosol Rotating Drum (R=50cm,t=3hr)",
    AxesLabel -> {"D(\[Mu])", "rpm", "h/r"}, PlotPoints -> 20];
```

MATHEMATICA file MossN(T,D).nb

```
rho = 1;
mu = 1.83*10^-4;
d = DD*10^-4;
omega = 2*Pi*rpm/60;
t = thour*3600;
CC = 1 + (2*.071*10^-4/d)(1.257 + .4*Exp[-1.1*d/(2*.071*10^-4)]);
tau = rho*d^2 CC/(18*mu);
p = Exp[-2*tau*t*omega^2];
h = If[Pi <=
      omega*t, (tau*980/omega)(1 +
         p^.5), (tau*980/omega)(1 - 2*Cos[omega*t]*p^.5 + p)^.5];
h1 = (r*r + h*h - r*r*p)/(2*h);
h2 = h - h1;
r = 50;
f1 = p;
f2 = (1 + p)/2 -
    1/Pi*(h1/r*(1 - (h1/r)^2)^.5 + ArcSin[h1/r] + h2/r*(p - (h2/r)^2)^.5 +
        p*ArcSin[h2/r]);
f3 = 0;
hrlow = 1 - p^0.5;
hrhigh = 1 + p^0.5;
rpm = .5;
g1 = Plot3D[
    If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001,
      48}, {DD, .01, 20},
    PlotLabel -> "Aerosol Rotating Drum (R=50cm,rpm=.5)",
    AxesLabel -> {"thour", "D(μ)", "N/No"}, PlotPoints -> 20];
g1 = Plot3D[h/r, {thour, 0.001, 48}, {DD, .01, 20},
    PlotLabel -> "Aerosol Rotating Drum (R=50cm,rpm=.5)",
    AxesLabel -> {"thour", "D(μ)", "h/r"}, PlotPoints -> 20];
rpm = 1;
g1 = Plot3D[
    If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001,
      48}, {DD, .01, 20},
    PlotLabel -> "Aerosol Rotating Drum (R=50cm,rpm=1)",
    AxesLabel -> {"thour", "D(μ)", "N/No"}, PlotPoints -> 20];
rpm = 2;
g1 = Plot3D[
    If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001,
```

```
    48}, {DD, .01, 20},
   PlotLabel -> "Aerosol Rotating Drum (R=50cm,rpm=2)",
   AxesLabel -> {"thour", "D(μ)", "N/No"}, PlotPoints -> 20];
rpm = 5;
g1 = Plot3D[
   If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001,
    48}, {DD, .01, 20},
   PlotLabel -> "Aerosol Rotating Drum (R=50cm,rpm=5)",
   AxesLabel -> {"thour", "D(μ)", "N/No"}, PlotPoints -> 20];
```

MATHEMATICA file MossN(T,RPM).nb

```
rho = 1;
mu = 1.83*10^-4;
d = DD*10^-4;
omega = 2*Pi*rpm/60;
t = thour*3600;
CC = 1 + (2*.071*10^-4/d)(1.257 + .4*Exp[-1.1*d/(2*.071*10^-4)]);
tau = rho*d^2 CC/(18*mu);
p = Exp[-2*tau*t*omega^2];
h = If[Pi <=
     omega*t, (tau*980/omega)(1 +
        p^.5), (tau*980/omega)(1 - 2*Cos[omega*t]*p^.5 + p)^.5];
h1 = (r*r + h*h - r*r*p)/(2*h);
h2 = h - h1;
r = 50;
f1 = p;
f2 = (1 + p)/2 -
    1/Pi*(h1/r*(1 - (h1/r)^2)^.5 + ArcSin[h1/r] + h2/r*(p - (h2/r)^2)^.5 +
        p*ArcSin[h2/r]);
f3 = 0;
hrlow = 1 - p^0.5;
hrhigh = 1 + p^0.5;
DD = 20;
g1 = Plot3D[
   If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.0001,
    50}, {rpm, .0001, 5},
   PlotLabel -> "Aerosol Rotating Drum (R=50cm,D=20μ)",
   AxesLabel -> {"thour", "rpm", "N/No"}, PlotPoints -> 30];
DD = 10;
g1 = Plot3D[
   If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.0001,
    50}, {rpm, .0001, 5},
   PlotLabel -> "Aerosol Rotating Drum (R=50cm,D=10μ)",
   AxesLabel -> {"thour", "rpm", "N/No"}, PlotPoints -> 30];
DD = 5;
```

```
gl = Plot3D[
    If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.0001,
      50}, {rpm, .0001, 5},
    PlotLabel -> "Aerosol Rotating Drum (R=50cm,D=5μ)",
    AxesLabel -> {"thour", "rpm", "N/No"}, PlotPoints -> 30];
DD = 2;
gl = Plot3D[
    If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.0001,
      50}, {rpm, .0001, 5},
    PlotLabel -> "Aerosol Rotating Drum (R=50cm,D=2μ)",
    AxesLabel -> {"thour", "rpm", "N/No"}, PlotPoints -> 30];
DD = 1;
gl = Plot3D[
    If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.0001,
      50}, {rpm, .0001, 5},
    PlotLabel -> "Aerosol Rotating Drum (R=50cm,D=1μ)",
    AxesLabel -> {"thour", "rpm", "N/No"}, PlotPoints -> 30];
DD = .5;
gl = Plot3D[
    If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.0001,
      50}, {rpm, .0001, 5},
    PlotLabel -> "Aerosol Rotating Drum (R=50cm,D=.5μ)",
    AxesLabel -> {"thour", "rpm", "N/No"}, PlotPoints -> 30];
gl = Plot3D[h/r, {thour, 0.0001, 50}, {rpm, .0001, 5},
    PlotLabel -> "Aerosol Rotating Drum (R=50cm,D=.5μ)",
    AxesLabel -> {"thour", "rpm", "h/r"}, PlotPoints -> 30];
```

MATHEMATICA file MossD=1.nb

```
rho = 1;
mu = 1.83*10^-4;
d = DD*10^-4;
omega = 2*Pi*rpm/60;
t = thour*3600;
CC = 1 + (2*.071*10^-4/d)(1.257 + .4*Exp[-1.1*d/(2*.071*10^-4)]);
tau = rho*d^2 CC/(18*mu);
p = Exp[-2*tau*t*omega^2];
h = If[Pi <=
      omega*t, (tau*980/omega)(1 +
        p^.5), (tau*980/omega)(1 - 2*Cos[omega*t]*p^.5 + p)^.5];
h1 = (r*r + h*h - r*r*p)/(2*h);
h2 = h - h1;
r = 50;
f1 = p;
f2 = (1 + p)/2 -
    1/Pi*(h1/r*(1 - (h1/r)^2)^.5 + ArcSin[h1/r] + h2/r*(p - (h2/r)^2)^.5 +
        p*ArcSin[h2/r]);
```

```
f3 = 0;
hrlow = 1 - p^0.5;
hrhigh = 1 + p^0.5;
rpm = .0000001;
DD = 1;
g0 = Plot[If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 50},
     PlotLabel -> "Rotating Drum (R=50cm,rpm=0,D=1\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 1;
g2 = Plot[If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 50},
     PlotLabel -> "Rotating Drum (R=50cm,rpm=1,D=1\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 2;
g3 = Plot[If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 50},
     PlotLabel -> "Rotating Drum (R=50cm,rpm=2,D=1\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 4;
g4 = Plot[If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 50},
     PlotLabel -> "Rotating Drum (R=50cm,rpm=4,D=1\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 6;
g1 = Plot[If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 50},
     PlotLabel -> "Rotating Drum (R=50cm,rpm=6,D=1\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 8;
g5 = Plot[If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 50},
     PlotLabel -> "Rotating Drum (R=50cm,rpm=8,D=1\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
Show[g0, g1, g2, g3, g4, g5, PlotRange -> {0, 1},
  PlotLabel -> "    Rotating Drum (rpm=0,1,2,4,6,8;D=1\[Mu])"];
```

MATHEMATICA file MossD=2.nb

```
rho = 1;
mu = 1.83*10^-4;
d = DD*10^-4;
omega = 2*Pi*rpm/60;
t = thour*3600;
CC = 1 + (2*.071*10^-4/d)(1.257 + .4*Exp[-1.1*d/(2*.071*10^-4)]);
tau = rho*d^2 CC/(18*mu);
p = Exp[-2*tau*t*omega^2];
h = If[Pi <=
      omega*t, (tau*980/omega)(1 +
         p^.5), (tau*980/omega)(1 - 2*Cos[omega*t]*p^.5 + p)^.5];
h1 = (r*r + h*h - r*r*p)/(2*h);
h2 = h - h1;
r = 50;
```

```
f1 = p;
f2 = (1 + p)/2 -
   1/Pi*(h1/r*(1 - (h1/r)^2)^.5 + ArcSin[h1/r] + h2/r*(p - (h2/r)^2)^.5 +
      p*ArcSin[h2/r]);
f3 = 0;
hrlow = 1 - p^0.5;
hrhigh = 1 + p^0.5;
rpm = .0000001;
DD = 2;
g0 = Plot[If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=0,D=2\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 1;
g2 = Plot[If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=1,D=2\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 2;
g3 = Plot[If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=2,D=2\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 4;
g4 = Plot[If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=4,D=2\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 6;
g1 = Plot[If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=6,D=2\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 8;
g5 = Plot[If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=8,D=2\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
Show[g0, g1, g2, g3, g4, g5, PlotRange -> {0, 1},
   PlotLabel -> "    Rotating Drum (rpm=0,1,2,4,6,8;D=2\[Mu])"];
```

MATHEMATICA file MossD=3.nb

```
rho = 1;
mu = 1.83*10^-4;
d = DD*10^-4;
omega = 2*Pi*rpm/60;
t = thour*3600;
CC = 1 + (2*.071*10^-4/d)(1.257 + .4*Exp[-1.1*d/(2*.071*10^-4)]);
tau = rho*d^2 CC/(18*mu);
p = Exp[-2*tau*t*omega^2];
h = If[Pi <=
```

```
    omega*t, (tau*980/omega)(1 +
        p^.5), (tau*980/omega)(1 - 2*Cos[omega*t]*p^.5 + p)^.5];
h1 = (r*r + h*h - r*r*p)/(2*h);
h2 = h - h1;
r = 50;
f1 = p;
f2 = (1 + p)/2 -
    1/Pi*(h1/r*(1 - (h1/r)^2)^.5 + ArcSin[h1/r] + h2/r*(p - (h2/r)^2)^.5 +
        p*ArcSin[h2/r]);
f3 = 0;
hrlow = 1 - p^0.5;
hrhigh = 1 + p^0.5;
rpm = .0000001;
DD = 3;
g0 = Plot[If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 50},
     PlotLabel -> "Rotating Drum (R=50cm,rpm=0,D=3\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 1;
g2 = Plot[If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 50},
     PlotLabel -> "Rotating Drum (R=50cm,rpm=1,D=3\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 2;
g3 = Plot[If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 50},
     PlotLabel -> "Rotating Drum (R=50cm,rpm=2,D=3\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 4;
g4 = Plot[If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 50},
     PlotLabel -> "Rotating Drum (R=50cm,rpm=4,D=3\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 6;
g1 = Plot[If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 50},
     PlotLabel -> "Rotating Drum (R=50cm,rpm=6,D=3\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 8;
g5 = Plot[If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 50},
     PlotLabel -> "Rotating Drum (R=50cm,rpm=8,D=3\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
Show[g0, g1, g2, g3, g4, g5, PlotRange -> {0, 1},
   PlotLabel -> "    Rotating Drum (rpm=0,1,2,4,6,8;D=3\[Mu])"];
```

MATHEMATICA file MossD=4.nb

```
rho = 1;
mu = 1.83*10^-4;
d = DD*10^-4;
omega = 2*Pi*rpm/60;
```

```
t = thour*3600;
CC = 1 + (2*.071*10^-4/d)(1.257 + .4*Exp[-1.1*d/(2*.071*10^-4)]);
tau = rho*d^2 CC/(18*mu);
p = Exp[-2*tau*t*omega^2];
h = If[Pi <=
     omega*t, (tau*980/omega)(1 +
        p^.5), (tau*980/omega)(1 - 2*Cos[omega*t]*p^.5 + p)^.5];
h1 = (r*r + h*h - r*r*p)/(2*h);
h2 = h - h1;
r = 50;
f1 = p;
f2 = (1 + p)/2 -
    1/Pi*(h1/r*(1 - (h1/r)^2)^.5 + ArcSin[h1/r] + h2/r*(p - (h2/r)^2)^.5 +
          p*ArcSin[h2/r]);
f3 = 0;
hrlow = 1 - p^0.5;
hrhigh = 1 + p^0.5;
rpm = .0000001;
DD = 4;
g0 = Plot[If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 25},
     PlotLabel -> "Rotating Drum (R=50cm,rpm=0,D=4\[Mu])",
     AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 1;
g2 = Plot[If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 25},
     PlotLabel -> "Rotating Drum (R=50cm,rpm=1,D=4\[Mu])",
     AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 2;
g3 = Plot[If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 25},
     PlotLabel -> "Rotating Drum (R=50cm,rpm=2,D=4\[Mu])",
     AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 4;
g4 = Plot[If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 25},
     PlotLabel -> "Rotating Drum (R=50cm,rpm=4,D=4\[Mu])",
     AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 6;
g1 = Plot[If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 25},
     PlotLabel -> "Rotating Drum (R=50cm,rpm=6,D=4\[Mu])",
     AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 8;
g5 = Plot[If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 25},
     PlotLabel -> "Rotating Drum (R=50cm,rpm=8,D=4\[Mu])",
     AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
Show[g0, g1, g2, g3, g4, g5, PlotRange -> {0, 1},
   PlotLabel -> "    Rotating Drum (rpm=0,1,2,4,6,8;D=4\[Mu])"];
```

MATHEMATICA file MossD=5.nb

```
rho = 1;
mu = 1.83*10^-4;
d = DD*10^-4;
omega = 2*Pi*rpm/60;
t = thour*3600;
CC = 1 + (2*.071*10^-4/d)(1.257 + .4*Exp[-1.1*d/(2*.071*10^-4)]);
tau = rho*d^2 CC/(18*mu);
p = Exp[-2*tau*t*omega^2];
h = If[Pi <=
      omega*t, (tau*980/omega)(1 +
          p^.5), (tau*980/omega)(1 - 2*Cos[omega*t]*p^.5 + p)^.5];
h1 = (r*r + h*h - r*r*p)/(2*h);
h2 = h - h1;
r = 50;
f1 = p;
f2 = (1 + p)/2 -
    1/Pi*(h1/r*(1 - (h1/r)^2)^.5 + ArcSin[h1/r] + h2/r*(p - (h2/r)^2)^.5 +
        p*ArcSin[h2/r]);
f3 = 0;
hrlow = 1 - p^0.5;
hrhigh = 1 + p^0.5;
rpm = .0000001;
DD = 5;
g0 = Plot[If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 25},
     PlotLabel -> "Rotating Drum (R=50cm,rpm=0,D=5\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 1;
g2 = Plot[If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 25},
     PlotLabel -> "Rotating Drum (R=50cm,rpm=1,D=5\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 2;
g3 = Plot[If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 25},
     PlotLabel -> "Rotating Drum (R=50cm,rpm=2,D=5\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 4;
g4 = Plot[If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 25},
     PlotLabel -> "Rotating Drum (R=50cm,rpm=4,D=5\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 6;
g1 = Plot[If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 25},
     PlotLabel -> "Rotating Drum (R=50cm,rpm=6,D=5\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 8;
g5 = Plot[If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 25},
     PlotLabel -> "Rotating Drum (R=50cm,rpm=8,D=5\[Mu])",
```

```
   AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
Show[g0, g1, g2, g3, g4, g5, PlotRange -> {0, 1},
  PlotLabel -> "   Rotating Drum (rpm=0,1,2,4,6,8;D=5\[Mu])"];
```

MATHEMATICA file MossD=6.nb

```
rho = 1;
mu = 1.83*10^-4;
d = DD*10^-4;
omega = 2*Pi*rpm/60;
t = thour*3600;
CC = 1 + (2*.071*10^-4/d)(1.257 + .4*Exp[-1.1*d/(2*.071*10^-4)]);
tau = rho*d^2 CC/(18*mu);
p = Exp[-2*tau*t*omega^2];
h = If[Pi <=
     omega*t, (tau*980/omega)(1 +
        p^.5), (tau*980/omega)(1 - 2*Cos[omega*t]*p^.5 + p)^.5];
h1 = (r*r + h*h - r*r*p)/(2*h);
h2 = h - h1;
r = 50;
f1 = p;
f2 = (1 + p)/2 -
    1/Pi*(h1/r*(1 - (h1/r)^2)^.5 + ArcSin[h1/r] + h2/r*(p - (h2/r)^2)^.5 +
        p*ArcSin[h2/r]);
f3 = 0;
hrlow = 1 - p^0.5;
hrhigh = 1 + p^0.5;
rpm = .0000001;
DD = 6;
g0 = Plot[If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 7},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=0,D=6\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 1;
g2 = Plot[If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 7},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=1,D=6\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 2;
g3 = Plot[If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 7},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=2,D=6\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 4;
g4 = Plot[If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 7},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=4,D=6\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 6;
g1 = Plot[If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 7},
```

```
    PlotLabel -> "Rotating Drum (R=50cm,rpm=6,D=6\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 8;
g5 = Plot[If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 7},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=8,D=6\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
Show[g0, g1, g2, g3, g4, g5, PlotRange -> {0, 1},
   PlotLabel -> "    Rotating Drum (rpm=0,1,2,4,6,8;D=6\[Mu])"];
```

MATHEMATICA file RatioGruelMoss(T,RPM).nb

```
rho = 1;
mu = 1.83*10^-4;
d = DD*10^-4;
omega = 2*Pi*rpm/60;
t = thour*3600;
CC = 1 + (2*.071*10^-4/d)(1.257 + .4*Exp[-1.1*d/(2*.071*10^-4)]);
tau = rho*d^2 CC/(18*mu);
p = Exp[-2*tau*t*omega^2];
h = If[Pi <=
      omega*t, (tau*980/omega)(1 +
         p^.5), (tau*980/omega)(1 - 2*Cos[omega*t]*p^.5 + p)^.5];
h1 = (r*r + h*h - r*r*p)/(2*h);
h2 = h - h1;
r = 50;
f1 = p;
f2 = (1 + p)/2 -
    1/Pi*(h1/r*(1 - (h1/r)^2)^.5 + ArcSin[h1/r] + h2/r*(p - (h2/r)^2)^.5 +
        p*ArcSin[h2/r]);
f3 = 0;
hrlow = 1 - p^0.5;
hrhigh = 1 + p^0.5;
r0 = tau 980/omega;
fg1 = If[thour <= 1/60/rpm,
    1 - (1 - (r - r0)^2/r^2)(thour/60/rpm)Exp[-2*tau*t*omega^2], (r -
          r0)^2 Exp[-2*tau*t*omega^2]/r^2];
fg2 = 0;
fg = If[r <= r0, fg2, fg1];
DD = 20;
g1 = Plot3D[
    If[h/r <= hrlow, fg/f1, If[hrhigh <= h/r, fg/f3, fg/f2]], {thour,
      0.0001, 50}, {rpm, .1, 5},
    PlotLabel -> "Aerosol Rotating Drum (R=50cm,D=20µ)",
    AxesLabel -> {"thour", "rpm", "Ng/Nm"}, PlotPoints -> 30];
DD = 10;
```

```
g1 = Plot3D[
   If[h/r <= hrlow, fg/f1, If[hrhigh <= h/r, fg/f3, fg/f2]], {thour,
     0.0001, 50}, {rpm, .1, 5},
   PlotLabel -> "Aerosol Rotating Drum (R=50cm,D=10µ)",
   AxesLabel -> {"thour", "rpm", "Ng/Nm"}, PlotPoints -> 30];
DD = 5;
g1 = Plot3D[
   If[h/r <= hrlow, fg/f1, If[hrhigh <= h/r, fg/f3, fg/f2]], {thour,
     0.0001, 50}, {rpm, .1, 5},
   PlotLabel -> "Aerosol Rotating Drum (R=50cm,D=5µ)",
   AxesLabel -> {"thour", "rpm", "Ng/Nm"}, PlotPoints -> 30];
DD = 2;
g1 = Plot3D[
   If[h/r <= hrlow, fg/f1, If[hrhigh <= h/r, fg/f3, fg/f2]], {thour,
     0.0001, 50}, {rpm, .1, 5},
   PlotLabel -> "Aerosol Rotating Drum (R=50cm,D=2µ)",
   AxesLabel -> {"thour", "rpm", "Ng/Nm"}, PlotPoints -> 30];
DD = 1;
g1 = Plot3D[
   If[h/r <= hrlow, fg/f1, If[hrhigh <= h/r, fg/f3, fg/f2]], {thour,
     0.0001, 50}, {rpm, .1, 5},
   PlotLabel -> "Aerosol Rotating Drum (R=50cm,D=1µ)",
   AxesLabel -> {"thour", "rpm", "Ng/Nm"}, PlotPoints -> 30];
DD = .5;
g1 = Plot3D[
   If[h/r <= hrlow, fg/f1, If[hrhigh <= h/r, fg/f3, fg/f2]], {thour,
     0.0001, 50}, {rpm, .1, 5},
   PlotLabel -> "Aerosol Rotating Drum (R=50cm,D=.5µ)",
   AxesLabel -> {"thour", "rpm", "Ng/Nm"}, PlotPoints -> 30];
```

MATHEMATICA file RatioGruelMoss(T,D).nb

```
rho = 1;
mu = 1.83*10^-4;
d = DD*10^-4;
omega = 2*Pi*rpm/60;
t = thour*3600;
CC = 1 + (2*.071*10^-4/d)(1.257 + .4*Exp[-1.1*d/(2*.071*10^-4)]);
tau = rho*d^2 CC/(18*mu);
p = Exp[-2*tau*t*omega^2];
h = If[Pi <=
      omega*t, (tau*980/omega)(1 +
        p^.5), (tau*980/omega)(1 - 2*Cos[omega*t]*p^.5 + p)^.5];
h1 = (r*r + h*h - r*r*p)/(2*h);
h2 = h - h1;
```

```
r = 50;
f1 = p;
f2 = (1 + p)/2 -
    1/Pi*(h1/r*(1 - (h1/r)^2)^.5 + ArcSin[h1/r] + h2/r*(p - (h2/r)^2)^.5 +
        p*ArcSin[h2/r]);
f3 = 0;
hrlow = 1 - p^0.5;
hrhigh = 1 + p^0.5;
r0 = tau 980/omega; fg1 =
  If[thour <= 1/60/rpm,
   1 - (1 - (r - r0)^2/r^2)(thour/60/rpm)Exp[-2*tau*t*omega^2], (r -
        r0)^2 Exp[-2*tau*t*omega^2]/r^2];
fg2 = 0;
fg = If[r <= r0, fg2, fg1];
rpm = .5;
g1 = Plot3D[
    If[h/r <= hrlow, fg/f1, If[hrhigh <= h/r, fg/f3, fg/f2]], {thour, 0.001,
      48}, {DD, .01, 20},
    PlotLabel -> "Aerosol Rotating Drum (R=50cm,rpm=.5)",
    AxesLabel -> {"thour", "D(µ)", "Ng/Nm"}, PlotPoints -> 20];
rpm = 1;
g1 = Plot3D[
    If[h/r <= hrlow, fg/f1, If[hrhigh <= h/r, fg/f3, fg/f2]], {thour, 0.001,
      48}, {DD, .01, 20},
    PlotLabel -> "Aerosol Rotating Drum (R=50cm,rpm=1)",
    AxesLabel -> {"thour", "D(µ)", "Ng/Nm"}, PlotPoints -> 20];
rpm = 2;
g1 = Plot3D[
    If[h/r <= hrlow, fg/f1, If[hrhigh <= h/r, fg/f3, fg/f2]], {thour, 0.001,
      48}, {DD, .01, 20},
    PlotLabel -> "Aerosol Rotating Drum (R=50cm,rpm=2)",
    AxesLabel -> {"thour", "D(µ)", "Ng/Nm"}, PlotPoints -> 20];
rpm = 5;
g1 = Plot3D[
    If[h/r <= hrlow, fg/f1, If[hrhigh <= h/r, fg/f3, fg/f2]], {thour, 0.001,
      48}, {DD, .01, 20},
    PlotLabel -> "Aerosol Rotating Drum (R=50cm,rpm=5)",
    AxesLabel -> {"thour", "D(µ)", "Ng/Nm"}, PlotPoints -> 20];
```

MATHEMATICA file RatioGruelMoss(D,RPM).nb

```
rho = 1;
mu = 1.83*10^-4;
d = DD*10^-4;
omega = 2*Pi*rpm/60;
```

```
t = thour*3600;
CC = 1 + (2*.071*10^-4/d)(1.257 + .4*Exp[-1.1*d/(2*.071*10^-4)]);
tau = rho*d^2 CC/(18*mu);
p = Exp[-2*tau*t*omega^2];
h = If[Pi <=
     omega*t, (tau*980/omega)(1 +
        p^.5), (tau*980/omega)(1 - 2*Cos[omega*t]*p^.5 + p)^.5];
h1 = (r*r + h*h - r*r*p)/(2*h);
h2 = h - h1;
r = 50;
f1 = p;
f2 = (1 + p)/2 -
    1/Pi*(h1/r*(1 - (h1/r)^2)^.5 + ArcSin[h1/r] + h2/r*(p - (h2/r)^2)^.5 +
        p*ArcSin[h2/r]);
f3 = 0;
hrlow = 1 - p^0.5;
hrhigh = 1 + p^0.5;
r0 = tau 980/omega;
fg1 = If[thour <= 1/60/rpm,
    1 - (1 - (r - r0)^2/r^2)(thour/60/rpm)Exp[-2*tau*t*omega^2], (r -
        r0)^2 Exp[-2*tau*t*omega^2]/r^2];
fg2 = 0;
fg = If[r <= r0, fg2, fg1];
thour = 3;
g1 = Plot3D[
    If[h/r <= hrlow, fg/f1, If[hrhigh <= h/r, fg/f3, fg/f2]], {DD, 0.01,
      20}, {rpm, .01, 5},
    PlotLabel -> "Aerosol Rotating Drum (R=50cm,t=3hr)",
    AxesLabel -> {"D(\[Mu])", "rpm", "Ng/Nm"}, PlotPoints -> 20];
thour = 24;
g1 = Plot3D[
    If[h/r <= hrlow, fg/f1, If[hrhigh <= h/r, fg/f3, fg/f2]], {DD, 0.01,
      20}, {rpm, .01, 2},
    PlotLabel -> "Aerosol Rotating Drum (R=50cm,t=24hr)",
    AxesLabel -> {"D(\[Mu])", "rpm", "Ng/Nm"}, PlotPoints -> 20];
thour = 96;
g1 = Plot3D[
    If[h/r <= hrlow, fg/f1, If[hrhigh <= h/r, fg/f3, fg/f2]], {DD, 0.01,
      20}, {rpm, .01, 2},
    PlotLabel -> "Aerosol Rotating Drum (R=50cm,t=96hr)",
    AxesLabel -> {"D(\[Mu])", "rpm", "Ng/Nm"}, PlotPoints -> 20];
```

MATHEMATICA file GruelN(T,D)&RPMopt.nb

```
rho = 1;
```

```
g = 980;
mu = 1.83*10^-4;
d = DD*10^-4;
omega = 2*Pi*rpm/60;
t = thour*3600;
CC = 1 + (2*.071*10^-4/d)(1.257 + .4*Exp[-1.1*d/(2*.071*10^-4)]);
tau = rho*d^2 CC/(18*mu);
p = Exp[-2*tau*t*omega^2];
r0 = tau 980/omega;
f1 = If[thour <= 1/60/rpm,
    1 - (1 - (r - r0)^2/r^2)(thour/60/rpm)Exp[-2*tau*t*omega^2], (r -
        r0)^2 Exp[-2*tau*t*omega^2]/r^2];
f2 = 0;
omegaopt = (g/2/r/t)^0.333333 + tau*g/3/r;
rpmopt = 60*omegaopt/2/Pi;
r = 50;
rpm = 0.5;
g1 = Plot3D[If[r <= r0, f2, f1], {DD, .001, 20}, {thour, 0.0001, 50},
    PlotLabel -> "Gruel Rotating Drum (R=50cm,rpm=0.5)",
    AxesLabel -> {"D(μ)", "thour", "N/No "}, PlotPoints -> 20];
rpm = 1;
g1 = Plot3D[If[r <= r0, f2, f1], {DD, .001, 20}, {thour, 0.0001, 50},
    PlotLabel -> "Gruel Rotating Drum (R=50cm,rpm=1)",
    AxesLabel -> {"D(μ)", "thour", "N/No "}, PlotPoints -> 20];
rpm = 2;
g1 = Plot3D[If[r <= r0, f2, f1], {DD, .001, 20}, {thour, 0.0001, 50},
    PlotLabel -> "Gruel Rotating Drum (R=50cm,rpm=2)",
    AxesLabel -> {"D(μ)", "thour", "N/No "}, PlotPoints -> 20];
g1 = Plot3D[rpmopt, {DD, .001, 20}, {thour, 0.0001, 50},
    PlotLabel -> "Optimum Rotation Rate (R=50cm)",
    AxesLabel -> {"D(μ)", "thour", "RPMopt "}, PlotPoints -> 20];
```

MATHEMATICA file GruelN(D,RPM).nb

```
rho = 1;
mu = 1.83*10^-4;
d = DD*10^-4;
omega = 2*Pi*rpm/60;
t = thour*3600;
CC = 1 + (2*.071*10^-4/d)(1.257 + .4*Exp[-1.1*d/(2*.071*10^-4)]);
tau = rho*d^2 CC/(18*mu);
p = Exp[-2*tau*t*omega^2];
r0 = tau 980/omega;
f1 = If[thour <= 1/60/rpm,
    1 - (1 - (r - r0)^2/r^2)(thour/60/rpm)Exp[-2*tau*t*omega^2], (r -
        r0)^2 Exp[-2*tau*t*omega^2]/r^2];
```

```
f2 = 0;
r = 50;
thour = 3;
g1 = Plot3D[If[r <= r0, f2, f1], {DD, 0.001, 20}, {rpm, .01, 5},
    PlotLabel -> "Gruel Rotating Drum (R=50cm,t=3hr)",
    AxesLabel -> {"D(\[Mu])", "rpm", "N/No"}, PlotPoints -> 20];
thour = 24;
g1 = Plot3D[If[r <= r0, f2, f1], {DD, 0.001, 20}, {rpm, .01, 5},
    PlotLabel -> "Gruel Rotating Drum (R=50cm,t=24hr)",
    AxesLabel -> {"D(\[Mu])", "rpm", "N/No"}, PlotPoints -> 20];
thour = 48;
g1 = Plot3D[If[r <= r0, f2, f1], {DD, 0.001, 20}, {rpm, .01, 5},
    PlotLabel -> "Gruel Rotating Drum (R=50cm,t=48hr)",
    AxesLabel -> {"D(\[Mu])", "rpm", "N/No"}, PlotPoints -> 20];
```

MATHEMATICA file GruelN(T,RPM).nb

```
rho = 1;
mu = 1.83*10^-4;
d = DD*10^-4;
omega = 2*Pi*rpm/60;
t = thour*3600;
CC = 1 + (2*.071*10^-4/d)(1.257 + .4*Exp[-1.1*d/(2*.071*10^-4)]);
tau = rho*d^2 CC/(18*mu);
p = Exp[-2*tau*t*omega^2];
r0 = tau 980/omega;
f1 = If[thour <= 1/60/rpm,
    1 - (1 - (r - r0)^2/r^2)(thour/60/rpm)Exp[-2*tau*t*omega^2], (r -
        r0)^2 Exp[-2*tau*t*omega^2]/r^2];
f2 = 0;
r = 50;
DD = 1;
g1 = Plot3D[If[r <= r0, f2, f1], {thour, .001, 50}, {rpm, .01, 5},
    PlotLabel -> "Gruel Rotating Drum (R=50cm,D=1μ)",
    AxesLabel -> {"thour", "rpm", "N/No"}, PlotPoints -> 30];
DD = 2;
g1 = Plot3D[If[r <= r0, f2, f1], {thour, .001, 50}, {rpm, .01, 5},
    PlotLabel -> "Gruel Rotating Drum (R=50cm,D=2μ)",
    AxesLabel -> {"thour", "rpm", "N/No"}, PlotPoints -> 30];
DD = 5;
g1 = Plot3D[If[r <= r0, f2, f1], {thour, .001, 50}, {rpm, .01, 5},
    PlotLabel -> "Gruel Rotating Drum (R=50cm,D=5μ)",
    AxesLabel -> {"thour", "rpm", "N/No"}, PlotPoints -> 30];
DD = 10;
g1 = Plot3D[If[r <= r0, f2, f1], {thour, .001, 50}, {rpm, .01, 5},
```

```
    PlotLabel -> "Gruel Rotating Drum (R=50cm,D=10μ)",
    AxesLabel -> {"thour", "rpm", "N/No"}, PlotPoints -> 30];
DD = 20;
g1 = Plot3D[If[r <= r0, f2, f1], {thour, .001, 50}, {rpm, .01, 5},
    PlotLabel -> "Gruel Rotating Drum (R=50cm,D=20μ)",
    AxesLabel -> {"thour", "rpm", "N/No"}, PlotPoints -> 30,
    PlotRange -> {0, 1}];
```

MATHEMATICA file StirredSettling.nb

```
rho = 1;
mu = 1.83*10^-4;
d = DD*10^-4;
t = thour*3600;
CC = 1 + (2*.071*10^-4/d)(1.257 + .4*Exp[-1.1*d/(2*.071*10^-4)]);
tau = rho*d^2 CC/(18*mu);
f = Exp[-2*tau*980*t/r/Pi];
r = 50;
DD = 1;
g0 = Plot[f, {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=0,D=1\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
DD = 2;
g1 = Plot[f, {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=0,D=2\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
DD = 3;
g2 = Plot[f, {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=0,D=3\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
DD = 4;
g3 = Plot[f, {thour, 0.001, 25},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=0,D=4\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
DD = 5;
g4 = Plot[f, {thour, 0.001, 25},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=0,D=5\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
Show[g0, g1, g2, g3, g4, PlotRange -> {{0, 25}, {0, 1}},
   PlotLabel -> "    Drum Stirred Settling (D=1,2,3,4,5\[Mu])"];
Show[g0, g1, g2, g3, g4, PlotRange -> {{0, 7}, {0, 1}},
   PlotLabel -> "    Drum Stirred Settling (D=1,2,3,4,5\[Mu])"];
```

MATHEMATICA file ElectrostaticDispersion.nb

```
th = 1/(4*pi*B*nu^2*ee^2*n0);
```

```
B = 1/(3*pi*eta*d);
ee = 4.8*10^-10;
eta = 1.83*10^-4;
d = dmicron*10^-4;
n0 = 1000;
thhour = th/3600;
nu = nufact*dmicron^2;
g1 = Plot3D[thhour, {nufact, 1, 10}, {dmicron, 1, 6},
    PlotLabel -> "Charged Cloud Halflife(n=1000",
    AxesLabel -> {"\[Nu]", "D(\[Mu])", "t(hr) "}, PlotPoints -> 20];
nu = nufact*dmicron; g1 =
  Plot3D[thhour, {nufact, 5, 15}, {dmicron, 1, 6},
   PlotLabel -> "Charged Cloud Halflife(n=1000",
   AxesLabel -> {"\[Nu]", "D(\[Mu])", "t(hr) "}, PlotPoints -> 20];
nu = nufact; g1 =
  Plot3D[thhour, {nufact, 10, 30}, {dmicron, 1, 6},
   PlotLabel -> "Charged Cloud Halflife(n=1000",
   AxesLabel -> {"\[Nu]", "D(\[Mu])", "t(hr) "}, PlotPoints -> 20];
```

MATHEMATICA file ElectrostaticDispersionFinal.nb

```
rho = 1;
mu = 1.83*10^-4;
d = DD*10^-4;
omega = 2*Pi*rpm/60;
t = thour*3600;
CC = 1 + (2*.071*10^-4/d)(1.257 + .4*Exp[-1.1*d/(2*.071*10^-4)]);
tau = rho*d^2 CC/(18*mu);
p = Exp[-2*tau*t*omega^2];
h = If[Pi <=
      omega*t, (tau*980/omega)(1 +
        p^.5), (tau*980/omega)(1 - 2*Cos[omega*t]*p^.5 + p)^.5];
h1 = (r*r + h*h - r*r*p)/(2*h);
h2 = h - h1;
r = 50;
f1 = p;
f2 = (1 + p)/2 -
    1/Pi*(h1/r*(1 - (h1/r)^2)^.5 + ArcSin[h1/r] + h2/r*(p - (h2/r)^2)^.5 +
        p*ArcSin[h2/r]);
f3 = 0;
hrlow = 1 - p^0.5;
hrhigh = 1 + p^0.5;
nn = 1/(Exp[2*tau*t*omega^2]*
        (1/nn0 + KK/(2*tau*omega^2)) - KK/(2*tau*omega^2));
KK = 6*10^-10 + 8*Pi*tau*qq^2/3/mm;
Feoverm = KK*nn;
mm = rho*Pi*d^3/6;
```

```
qq = ncharge*4.8*10^-10;
ncharge = 10*DD^2;
nn0 = 1000;
ff = nn/nn0;
rpm = .0000001;
DD = 1;
g1 = Plot[If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 50},
     PlotLabel -> "Rotating Drum (R=50cm,rpm=0,D=1\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 1;
g2 = Plot[f1, {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=1,D=1\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 2;
g3 = Plot[f1, {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=2,D=1\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 4;
g4 = Plot[f1, {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=4,D=1\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 6;
g5 = Plot[f1, {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=6,D=1\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 8;
g6 = Plot[f1, {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=8,D=1\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 1;
g7 = Plot[ff, {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=1,D=1\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 2;
g8 = Plot[ff, {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=2,D=1\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 4;
g9 = Plot[ff, {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=4,D=1\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 6;
g10 = Plot[ff, {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=6,D=1\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 8;
```

```
g11 = Plot[ff, {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=8,D=1\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = .0000001;
DD = 2;
g12 = Plot[
    If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=0,D=2\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 1;
g22 = Plot[f1, {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=1,D=2\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 2;
g32 = Plot[f1, {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=2,D=2\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 4;
g42 = Plot[f1, {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=4,D=2\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 6;
g52 = Plot[f1, {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=6,D=2\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 8;
g62 = Plot[f1, {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=8,D=2\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 1;
g72 = Plot[ff, {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=1,D=2\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 2;
g82 = Plot[ff, {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=2,D=2\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 4;
g92 = Plot[ff, {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=4,D=2\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 6;
g102 = Plot[ff, {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=6,D=2\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 8;
```

```
g112 = Plot[ff, {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=8,D=2\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = .0000001;
DD = 3;
g13 = Plot[
    If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=0,D=3\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 1;
g23 = Plot[f1, {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=1,D=3\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 2;
g33 = Plot[f1, {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=2,D=3\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 4;
g43 = Plot[f1, {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=4,D=3\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 6;
g53 = Plot[f1, {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=6,D=3\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 8;
g63 = Plot[f1, {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=8,D=3\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 1;
g73 = Plot[ff, {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=1,D=3\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 2;
g83 = Plot[ff, {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=2,D=3\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 4;
g93 = Plot[ff, {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=4,D=3\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 6;
g103 = Plot[ff, {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=6,D=3\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 8;
```

```
g113 = Plot[ff, {thour, 0.001, 50},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=8,D=3\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = .0000001;
DD = 4;
g14 = Plot[
    If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 25},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=0,D=4\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 1;
g24 = Plot[f1, {thour, 0.001, 25},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=1,D=4\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 2;
g34 = Plot[f1, {thour, 0.001, 25},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=2,D=4\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 4;
g44 = Plot[f1, {thour, 0.001, 25},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=4,D=4\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 6;
g54 = Plot[f1, {thour, 0.001, 25},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=6,D=4\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 8;
g64 = Plot[f1, {thour, 0.001, 25},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=8,D=4\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 1;
g74 = Plot[ff, {thour, 0.001, 25},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=1,D=4\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 2;
g84 = Plot[ff, {thour, 0.001, 25},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=2,D=4\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 4;
g94 = Plot[ff, {thour, 0.001, 25},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=4,D=4\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 6;
g104 = Plot[ff, {thour, 0.001, 25},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=6,D=4\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 8;
```

```
g114 = Plot[ff, {thour, 0.001, 25},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=8,D=4\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = .0000001;
DD = 5;
g15 = Plot[
    If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 25},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=0,D=5\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 1;
g25 = Plot[f1, {thour, 0.001, 25},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=1,D=5\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 2;
g35 = Plot[f1, {thour, 0.001, 25},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=2,D=5\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 4;
g45 = Plot[f1, {thour, 0.001, 25},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=4,D=5\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 6;
g55 = Plot[f1, {thour, 0.001, 25},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=6,D=5\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 8;
g65 = Plot[f1, {thour, 0.001, 25},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=8,D=5\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 1;
g75 = Plot[ff, {thour, 0.001, 25},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=1,D=5\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 2;
g85 = Plot[ff, {thour, 0.001, 25},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=2,D=5\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 4;
g95 = Plot[ff, {thour, 0.001, 25},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=4,D=5\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 6;
g105 = Plot[ff, {thour, 0.001, 25},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=6,D=5\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 8;
```

```
g115 = Plot[ff, {thour, 0.001, 25},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=8,D=5\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = .0000001;
DD = 6;
g16 = Plot[If[h/r <= hrlow, f1, If[hrhigh <= h/r, f3, f2]], {thour, 0.001, 7},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=0,D=6\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 1;
g26 = Plot[f1, {thour, 0.001, 7},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=1,D=6\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 2;
g36 = Plot[f1, {thour, 0.001, 7},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=2,D=6\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 4;
g46 = Plot[f1, {thour, 0.001, 7},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=4,D=6\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 6;
g56 = Plot[f1, {thour, 0.001, 7},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=6,D=6\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 8;
g66 = Plot[f1, {thour, 0.001, 7},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=8,D=6\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 1;
g76 = Plot[ff, {thour, 0.001, 7},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=1,D=6\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 2;
g86 = Plot[ff, {thour, 0.001, 7},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=2,D=6\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 4;
g96 = Plot[ff, {thour, 0.001, 7},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=4,D=6\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 6;
g106 = Plot[ff, {thour, 0.001, 7},
    PlotLabel -> "Rotating Drum (R=50cm,rpm=6,D=6\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
rpm = 8;
g116 = Plot[ff, {thour, 0.001, 7},
```

```
    PlotLabel -> "Rotating Drum (R=50cm,rpm=8,D=6\[Mu])",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
Plot[KK, {DD, 1, 10},
  PlotLabel -> "    Electrostatic Dispersion(Ncharge=10*D^2)",
  AxesLabel -> {"D(\[Mu])", "Ke  "}, PlotRange -> {0, 10^-6},
  PlotPoints -> 20];
Show[g1, g2, g3, g4, g5, g6, PlotRange -> {0, 1},
  PlotLabel -> "  Moss Theory (rpm=0,1,2,4,6,8;D=1\[Mu])"];
Show[g7, g8, g9, g10, g11, PlotRange -> {0, 1},
  PlotLabel -> "  Ncharge=10*D^2 (rpm=1,2,4,6,8;D=1\[Mu])"];
Show[g12, g22, g32, g42, g52, g62, PlotRange -> {0, 1},
  PlotLabel -> "  Moss Theory (rpm=0,1,2,4,6,8;D=2\[Mu])"];
Show[g72, g82, g92, g102, g112, PlotRange -> {0, 1},
  PlotLabel -> "  Ncharge=10*D^2 (rpm=1,2,4,6,8;D=2\[Mu])"];
Show[g13, g23, g33, g43, g53, g63, PlotRange -> {0, 1},
  PlotLabel -> "  Moss Theory (rpm=0,1,2,4,6,8;D=3\[Mu])"];
Show[g73, g83, g93, g103, g113, PlotRange -> {0, 1},
  PlotLabel -> "  Ncharge=10*D^2 (rpm=1,2,4,6,8;D=3\[Mu])"];
Show[g14, g24, g34, g44, g54, g64, PlotRange -> {0, 1},
  PlotLabel -> "  Moss Theory (rpm=0,1,2,4,6,8;D=4\[Mu])"];
Show[g74, g84, g94, g104, g114, PlotRange -> {0, 1},
  PlotLabel -> "  Ncharge=10*D^2 (rpm=1,2,4,6,8;D=4\[Mu])"];
Show[g15, g25, g35, g45, g55, g65, PlotRange -> {0, 1},
  PlotLabel -> "  Moss Theory (rpm=0,1,2,4,6,8;D=5\[Mu])"];
Show[g75, g85, g95, g105, g115, PlotRange -> {0, 1},
  PlotLabel -> "  Ncharge=10*D^2 (rpm=1,2,4,6,8;D=5\[Mu])"];
Show[g16, g26, g36, g46, g56, g66, PlotRange -> {0, 1},
  PlotLabel -> "  Moss Theory (rpm=0,1,2,4,6,8;D=6\[Mu])"];
Show[g76, g86, g96, g106, g116, PlotRange -> {0, 1},
  PlotLabel -> "  Ncharge=10*D^2 (rpm=1,2,4,6,8;D=6\[Mu])"];
```

MATHEMATICA file RadiusOfGyration.nb

```
(*using equivalent aerodynamic diameter so density = 1*)
rho = 1;
mu = 1.83*10^-4;
d = DD*10^-4;
omega = 2*Pi*rpm/60;
t = thour*3600;
CC = 1 + (2*.071*10^-4/d)(1.257 + .4*Exp[-1.1*d/(2*.071*10^-4)]);
tau = rho*d^2 CC/(18*mu);
p = Exp[-2*tau*t*omega^2];
r0 = tau 980/omega;
r = 50;
g1 = Plot3D[r0, {DD, .001, 10}, {rpm, 1, 8},
    PlotLabel -> "Radius of Gyration r0(cm)",
    AxesLabel -> {"D(\[Mu])", "rpm", "r0"}, PlotPoints -> 30];
```

```
Show[%, PlotRange -> {0, 3}];
volfractleft = If[r0 > r, 0, (r - r0)^2/r^2];
g1 = Plot3D[volfractleft, {DD, .001, 10}, {rpm, 1, 8},
    PlotLabel -> "N/N0 After One Revolution; R=50cm",
    AxesLabel -> {"D(\[Mu])", "rpm", "N/N0 "}, PlotPoints -> 30];
Show[%, PlotRange -> {0.85, 1}];
```

MATHEMATICA file RatioKNE0overKeq0.nb

```
rho = 1;
mu = 1.83*10^-4;
d = DD*10^-4;
omega = 2*Pi*rpm/60;
t = thour*3600;
CC = 1 + (2*.071*10^-4/d)(1.257 + .4*Exp[-1.1*d/(2*.071*10^-4)]);
tau = rho*d^2 CC/(18*mu);
r = 50;
nn = 1/(Exp[2*tau*t*omega^2]*
        (1/nn0 + KK/(2*tau*omega^2)) - KK/(2*tau*omega^2));
(*KK = 8*Pi*tau*qpart*qcloudavg/3/mm;
  mm = rho*Pi*d^3/6;
  qpart = ncharge*4.8*10^-10;
  ncharge = 10*DD^2;
  qcloudavg = 10*DDsquaredmean;
  DDsquaredmean = 9;*)
 ff = nn/nn0;
 ffoverff0 = ff/Exp[-2*tau*t*omega^2];
 thour = 48;
 KK = 0;
 nn0 = 1000;
 g1 = Plot3D[ff, {DD, 1, 10}, {rpm, 1, 8},
     PlotLabel -> "Kcoag + Kelec = 0 ;(No=1000,t=48hr)",
     AxesLabel -> {"D(\[Mu])", "rpm", "N/No"}, PlotPoints -> 20];
 KK = 10^-9;
 g1 = Plot3D[ff, {DD, 1, 10}, {rpm, 1, 8},
     PlotLabel -> "Kcoag + Kelec = 10^-9 ;(No=1000,t=48hr)",
     AxesLabel -> {"D(\[Mu])", "rpm", "N/No"}, PlotPoints -> 20];
 g1 = Plot3D[ffoverff0, {DD, 1, 10}, {rpm, 1, 8},
     PlotLabel -> "N1(Kcoag+Kelec=10^-9)/N2(K=0); No=1000,t=48hr",
     AxesLabel -> {"D(\[Mu])", "rpm", "N1/N2 "}, PlotPoints -> 20];
 nn0 = 500;
 g1 = Plot3D[ffoverff0, {DD, 1, 10}, {rpm, 1, 8},
     PlotLabel -> "N1(Kcoag+Kelec=10^-9)/N2(K=0); No=500,t=48hr",
     AxesLabel -> {"D(\[Mu])", "rpm", "N1/N2 "}, PlotPoints -> 20];
 nn0 = 1000;
 KK = 10^-8;
 g1 = Plot3D[ff, {DD, 1, 10}, {rpm, 1, 8},
```

```
    PlotLabel -> "Kcoag + Kelec = 10^-8 ;(No=1000,t=48hr)",
    AxesLabel -> {"D(\[Mu])", "rpm", "N/No"}, PlotPoints -> 20];
g1 = Plot3D[ffoverff0, {DD, 1, 10}, {rpm, 1, 8},
    PlotLabel -> "N1(Kcoag+Kelec=10^-8)/N2(K=0); No=1000,t=48hr",
    AxesLabel -> {"D(\[Mu])", "rpm", "N1/N2 "}, PlotPoints -> 20];
nn0 = 500;
g1 = Plot3D[ffoverff0, {DD, 1, 10}, {rpm, 1, 8},
    PlotLabel -> "N1(Kcoag+Kelec=10^-8)/N2(K=0); No=500,t=48hr",
    AxesLabel -> {"D(\[Mu])", "rpm", "N1/N2 "}, PlotPoints -> 20];
nn0 = 700;
KK = 10^-9;
g1 = Plot3D[ffoverff0, {DD, 1, 10}, {rpm, 1, 8},
    PlotLabel -> "N1/N2\[Congruent]N(K=10^-9)/N(K=0); No=700,t=48hr",
    AxesLabel -> {"D(\[Mu])", "rpm", "N1/N2 "}, PlotRange -> {.85, 1},
    PlotPoints -> 20];
nn0 = 700;
KK = 10^-8;
g1 = Plot3D[ffoverff0, {DD, 1, 10}, {rpm, 1, 8},
    PlotLabel -> "N1/N2\[Congruent]N(K=10^-8)/N(K=0); No=700,t=48hr",
    AxesLabel -> {"D(\[Mu])", "rpm", "N1/N2 "}, PlotRange -> {.4, 1},
    PlotPoints -> 20];
thour = 24;
nn0 = 700;
KK = 8*Pi*tau*qpart*qcloudavg/3/mm;
mm = rho*Pi*d^3/6;
qpart = ncharge*4.8*10^-10;
ncharge = 10*DD^2;
qcloudavg = 10*DDsquaredmean*4.8*10^-10; (*depends on dist funct*)
DDsquaredmean = 9;
g1 = Plot3D[ffoverff0, {DD, 1, 10}, {rpm, 1, 8}, PlotLabel -> "\!\(\*
StyleBox[\"N1\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"/\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"N2\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"\[Congruent]\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"N\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"(\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"K\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"(\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"Ncharge\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"=\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"10\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"*\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"D\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"^\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"2\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\")\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
```

```
StyleBox[\"^\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"N\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"(\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"K\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"=\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"0\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\")\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\";\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\" \",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"No\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"=\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"700\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\",\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"t\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"=\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"24\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"hr\",\n\"SmallText\",\nFontSize->10]\)",
   AxesLabel -> {"D(\[Mu])", "rpm", "N1/N2 "}, PlotPoints -> 20];
g1 = Plot3D[KK, {DD, 1, 10}, {rpm, 1, 8},
    PlotLabel -> "Kelec with Ncharge=10*D^2; No=700,t=24hr",
    AxesLabel -> {"D(\[Mu])", "rpm", "K "}, PlotPoints -> 20];
```

MATHEMATICA file DiffusionDeposition.nb

```
t = thour*3600;
r = 50;
f = (6/Pi^2)Sum[Exp[-Diffcoef*Pi^2*i^2*t/r^2]/i^2, {i, 1, 99}];
Diffcoef = 10^-1;
g1 = Plot[f, {thour, 0.001, 50},
    PlotLabel -> "Diffusion Solution (R=50cm,DC=10^-1cm^2/s)",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
Diffcoef = 10^-2;
g2 = Plot[f, {thour, 0.001, 50},
    PlotLabel -> "Diffusion Solution (R=50cm,DC=10^-2cm^2/s)",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
Diffcoef = 10^-3;
g3 = Plot[f, {thour, 0.001, 50},
    PlotLabel -> "Diffusion Solution (R=50cm,DC=10^-3cm^2/s)",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
Diffcoef = 10^-4;
g4 = Plot[f, {thour, 0.001, 50},
    PlotLabel -> "Diffusion Solution (R=50cm,DC=10^-4cm^2/s)",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
Diffcoef = 10^-5;
g5 = Plot[f, {thour, 0.001, 50},
    PlotLabel -> "Diffusion Solution (R=50cm,DC=10^-5cm^2/s)",
    AxesLabel -> {"thour", "N/No"}, PlotPoints -> 20];
```

```
Show[g1, g2, g3, g4, g5, PlotRange -> {{0, 50}, {0, 1}}, PlotLabel -> "\!\(\*
StyleBox[\"      \",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"Diffusion\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\" \",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"Loss\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\";\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\" \",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"R\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"=\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"50\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"cm\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\",\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"-\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"Ln\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"(\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"D\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\")\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"=\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"1\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\",\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"2\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\",\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"3\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\",\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"4\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\",\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"5\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"cm\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"^\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"2\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"/\",\n\"SmallText\",\nFontSize->10]\)\!\(\*
StyleBox[\"s\",\n\"SmallText\",\nFontSize->10]\)"];
```

MATHEMATICA file DiffusionDepositionSurf.nb

```
rho = 1;
mu = 1.83*10^-4;
d = DD*10^-4;
omega = 2*Pi*rpm/60;
t = thour*3600;
CC = 1 + (2*.071*10^-4/d)(1.257 + .4*Exp[-1.1*d/(2*.071*10^-4)]);
tau = rho*d^2 CC/(18*mu);
r = 50;
b = r;
vcent = tau*omega^2*r;
f = (1 - Exp[-vcent*b/Diffcoef]);
DD = 2;
```

```
rpm = 4;
Diffcoef = 1;
g1 = Plot3D[f, {DD, 0.001, 10}, {rpm, 1, 8},
    PlotLabel -> "Deposition without/with diffusion; D=1cm^2/s",
    AxesLabel -> {"D(\[Mu])", "rpm", "WO/W"}, PlotPoints -> 20];
Diffcoef = 10^-1;
g1 = Plot3D[f, {DD, 0.001, 10}, {rpm, 1, 8},
    PlotLabel -> "Deposition without/with diffusion; D=.1cm^2/s",
    AxesLabel -> {"D(\[Mu])", "rpm", "WO/W"}, PlotPoints -> 20];
Diffcoef = 10^-2;
g1 = Plot3D[f, {DD, 0.001, 10}, {rpm, 1, 8},
    PlotLabel -> "Deposition without/with diffusion; D=.01cm^2/s",
    AxesLabel -> {"D(\[Mu])", "rpm", "WO/W"}, PlotPoints -> 20];
```